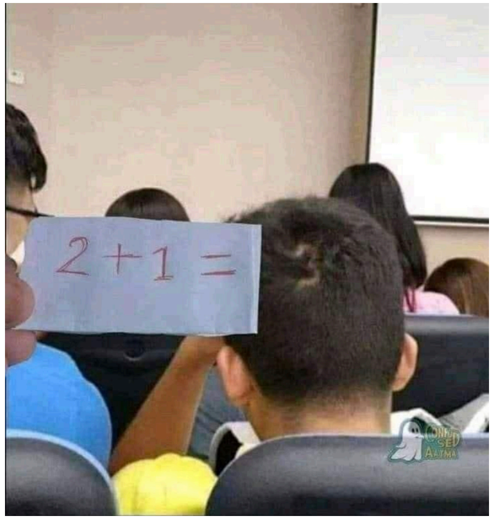
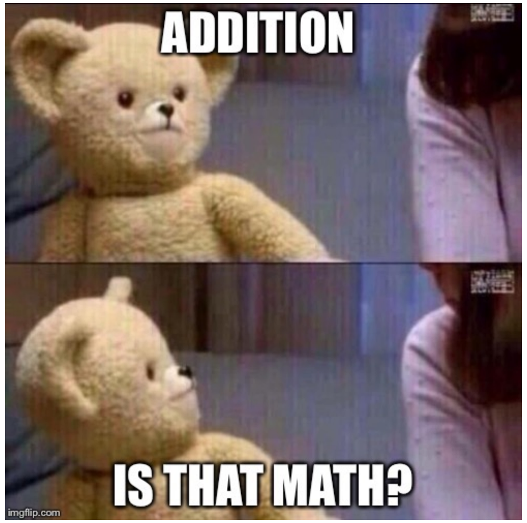
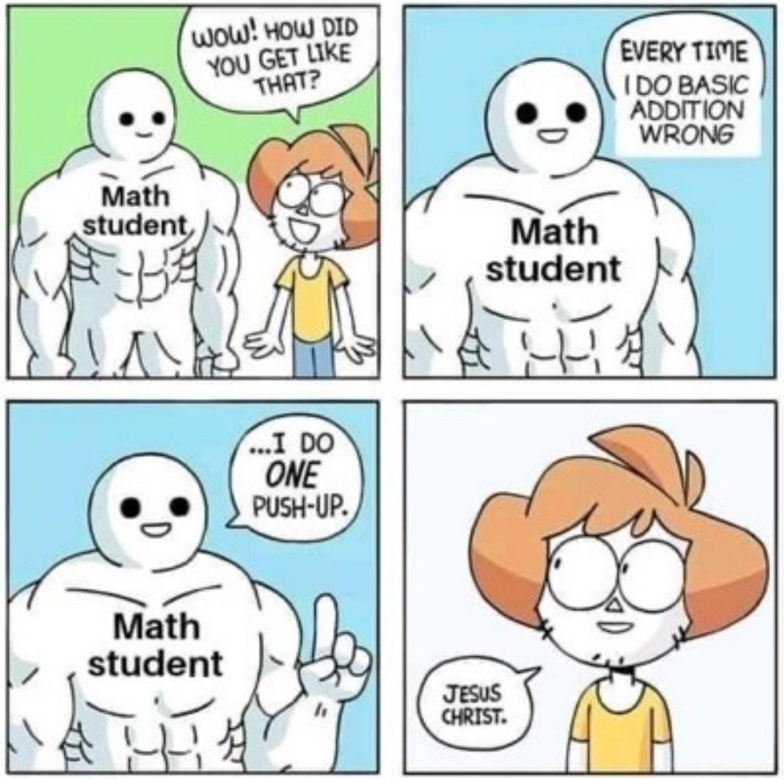
Basic Addition







Example 1 $15 + 24$

Method

We work from right to left and work on one column at a time

←

Step 1: Work on the column furthest to the right (add the digits)

Step 2: Work on next column to the left (add the digits)

If we had a bigger number (i.e. more digits) we keep going from right to left column by column until we run out of columns

step 2 do this next

step 1 start here

$$\begin{array}{r} 1\;5 \\ +\;2\;4 \\ \hline 3\;9 \end{array}$$

Tip:
It doesn't matter whether we put 15 or 24 on the top since adding in any order gives the same result

What happens if the digits of one of of the columns add up to more than 9 i.e. if any of our column additions give a two-digit number ? We will see how to deal with this on the next page.

Example 2 $\boxed{16 + 38}$

This example below has shown the stages and steps to explain (follow the step numbers to understand the flow). You should be able to do the final column on the right straight away once you get good. The examples on the next page are shown with one example only.

step ② step ①

$$\begin{array}{r} 1\ 6 \\ +\ 2\ 8 \\ \hline \underbrace{14} \\ \hline \end{array}$$

It is not ok to write a two-digit number here

step ③ Instead, we must "carry" the 1 to the next calculation ⟹

we bring the first digit up to join the next calculation

step ⑤ step ④

$$\begin{array}{r} {}^{1}1\ 6 \\ +\ 3\ 8 \\ \hline 4 \\ \hline \end{array}$$

step ⑥ This gives ⟹

step ⑧ step ⑦

$$\begin{array}{r} {}^{1}1\ 6 \\ +\ 3\ 8 \\ \hline 5\ 4 \\ \hline \end{array}$$

Further Examples

Example 3

63 + 82

$$\begin{array}{r} 63 \\ +\ 82 \\ \hline 145 \\ \hline \end{array}$$

Note It is ok here to write two-digits here since it is our final calculation.

Example 4

426 + 395

$$\begin{array}{r} {}^{1}4\,{}^{1}2\,6 \\ +\ 3\,9\,5 \\ \hline 8\,2\,1 \\ \hline \end{array}$$

4+3+1 =8

2+9+1 =12 bring the 1 up

11 bring the 1 up

Example 5

435 + 977

$$\begin{array}{r} {}^{1}4\,{}^{1}3\,5 \\ +\ 9\,7\,7 \\ \hline 1\,4\,1\,2 \\ \hline \end{array}$$

4+9+1 =14. We don't need to bring the 1 up since we are done

3+7+1 =11 bring the 1 up

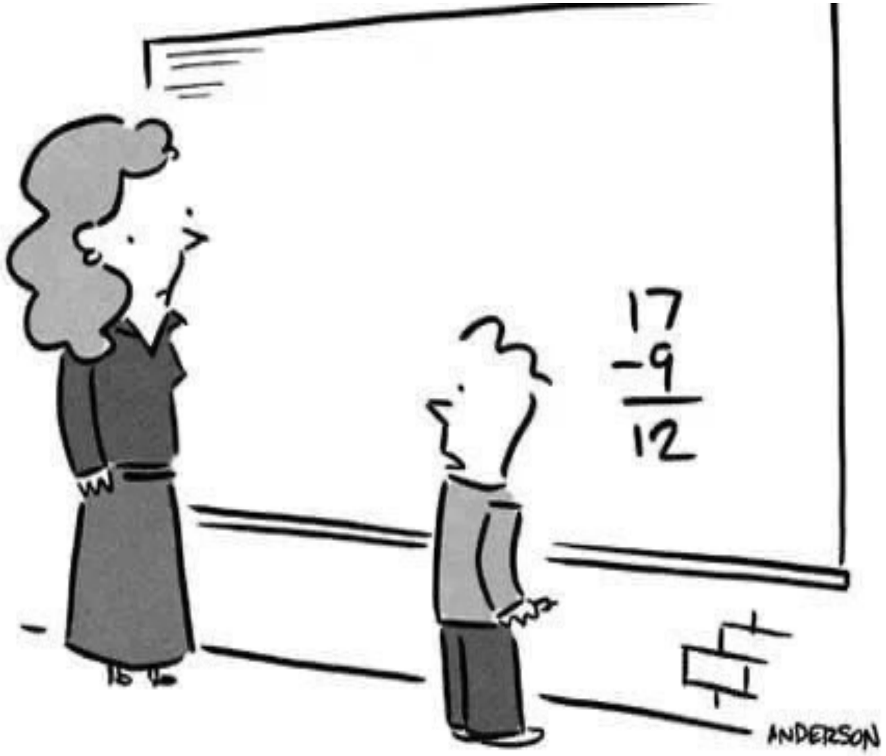
12 bring the 1 up

What happens if some of the digits are missing? Fill in any gaps with zeros and add as normal

$213 + 92$

$$
\begin{array}{r}
{}^{1}2\;1\;3 \\
+\;0\;9\;2 \\
\hline
3\;0\;5 \\
\hline
\end{array}
$$

Basic Subtraction



"I know it's wrong, I'm just waiting for the autocorrect."

Example 1 $\boxed{85 - 24}$

Method

Step 1: Work on the column furthest to the right (subtract the digits)

Step 2: Work on next column to the left (subtract the digits)

If we had a bigger number we keep going from right to left column by column until we run out of columns

←

② do this next ① start here

$$\begin{array}{r} 85 \\ -\ 24 \\ \hline 61 \\ \hline \end{array}$$

Unlike with addition, we must put the first number (85) on the top since subtraction in any order does not give the same result. We can't change the order of the question. For example:

$$4 - 2 = 2$$

but

$$2 - 4 = -2$$

What happens if the digit at the bottom is greater than the digit on the top in any column? We will see how to deal with this on the next page.

Example 2 $\boxed{85 - 37}$

Method

Step 1: borrow (add) a 10 since 5 is less than 7

Step 2: because we had to borrow in the first column (in step 1) we must steal from the next column (subtract) a 1

2 do this next

1 start here

$$\begin{array}{rcc} & {}^{7}\cancel{8} & \cancel{5}^{15} \\ - & 3 & 7 \\ \hline & 4 & 8 \\ \hline \end{array}$$

This is different to the last example. Why?
For each calculation we always need a bigger number on top. Here we do not have that for the pink calculation (7 is bigger than 5), so we need to **borrow** and steal. We always borrow 10 (add 10) for the first calculation and steal 1 (subtract 1) for the next calculation.

Example 3 435 – 269

Method

borrow (add) a 10
steal (subtract) a 1

This time we have to repeat the process:
borrow (add) a 10
steal (subtract) a 1

$$\begin{array}{rccc} & & 12 & \\ & 3 & \cancel{2} & 15 \\ & \cancel{4} & \cancel{3} & \cancel{5} \\ - & 2 & 6 & 9 \\ \hline & 1 & 6 & 6 \\ \hline \end{array}$$

This is harder that the last example. Why?
Since we have to borrow and steal __TWICE:__
For each calculation we always need a bigger number on top. Here we do not have that for the pink calculation **AND** the blue calculation, so we need to borrow and steal.

Example 4 $\boxed{202 - 54}$

This is harder than the last example since we are dealing with a 0 when we steal which is a little more confusing:

Method 1

We procced as usual, but here we need to take 1 away from 0.
When we take away 1 from 0 we are basically taking 1 away from 10 and therefore we turn the 0 into a 9. When we make a 0 and 9, we then ALSO AUTOMATICALLY make the next number 1 less.

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again (since we made a 0 a 9)

$$
\begin{array}{rccc}
 & 1 & 9 & 12 \\
 & \cancel{2} & \cancel{0} & 2 \\
- & & 5 & 4 \\
\hline
 & 1 & 4 & 8 \\
\hline
\end{array}
$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 20 to get 19

$$
\begin{array}{rccc}
 & 1 & 9 & 12 \\
 & \cancel{2} & \cancel{0} & 2 \\
- & & 5 & 4 \\
\hline
 & 1 & 4 & 8 \\
\hline
\end{array}
$$

Example 5 $3400 - 2246$

Method 1

We take away 1 from 0 we are basically taking 1 away from 10. We have to ALSO make the next number 1 less each time we change a 0 into a 9 and hence we and do it again

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again (since we made a 0 a 9)

$$\begin{array}{rcccc} & & 3 & 9 & 10 \\ & 3 & \cancel{4} & \cancel{0} & 0 \\ - & 2 & 2 & 4 & \cancel{6} \\ \hline & 1 & 1 & 5 & 4 \end{array}$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 40 to get 39

$$\begin{array}{rcccc} & & 3 & 9 & 10 \\ & 3 & \cancel{4} & \cancel{0} & \cancel{0} \\ - & 2 & 2 & 4 & 6 \\ \hline & 1 & 1 & 5 & 4 \end{array}$$

Example 6 $\boxed{3400 - 2746}$

This is harder that the last example since we borrow and steal twice:

Method 1

We take away 1 from 0 we are basically taking 1 away from 10. We have to ALSO make the next number 1 less each time we change a 0 into a 9 and hence we and do it again

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again

We repeat the process:
borrow (add) a 10
steal (subtract) a 1 again

$$\begin{array}{rcccc}
 & & 13 & & \\
 & 2 & \cancel{3} & 9 & 10 \\
 & \cancel{3} & \cancel{4} & \cancel{0} & \cancel{0} \\
- & 2 & 7 & 4 & 6 \\
\hline
 & 0 & 6 & 5 & 4 \\
\hline
\end{array}$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 40

$$\begin{array}{rcccc}
 & & 13 & & \\
 & 2 & \cancel{3} & 9 & 10 \\
 & \cancel{3} & \cancel{4} & \cancel{0} & \cancel{0} \\
- & 2 & 7 & 4 & 6 \\
\hline
 & 0 & 6 & 5 & 4 \\
\hline
\end{array}$$

Example 7 $\boxed{39000 - 26453}$

This is harder than the last example since we have successive 0's. Remember that with 0's we keep going:

Method 1

We take away 1 from 0 we are basically taking 1 away from 10. We have to ALSO make the next number 1 less each time we change a 0 into a 9 and hence we and do it again

Method

- borrow (add) a 10
- steal (subtract) a 1
- steal (subtract) a 1 again
- steal (subtract) a 1 again

$$
\begin{array}{cccccc}
 & & 8 & 9 & 9 & 10 \\
 & 3 & \cancel{9} & \cancel{0} & \cancel{0} & \cancel{0} \\
- & 2 & 6 & 4 & 5 & 3 \\
\hline
 & 1 & 2 & 5 & 4 & 7 \\
\hline
\end{array}
$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 900

$$
\begin{array}{cccccc}
 & & 8 & 9 & 9 & 10 \\
 & 3 & \cancel{9} & \cancel{0} & \cancel{0} & \cancel{0} \\
- & 2 & 6 & 4 & 5 & 3 \\
\hline
 & 1 & 2 & 5 & 4 & 7 \\
\hline
\end{array}
$$

Example 8 $80800 - 56722$

Method 1

Note: This zero did not becomes a 9, since we were done after the 8 became a 7 and we start the process of **borrowing** and stealing again

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again

We repeat the process:
borrow (add) a 10
steal (subtract) a 1

	7	10	7	9	10
	$\cancel{8}$	$\cancel{0}$	$\cancel{8}$	$\cancel{0}$	0
−	5	6	7	2	2
	2	4	0	7	8

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 80

7	10	7	9	10
8	$\cancel{0}$	$\cancel{8}$	$\cancel{0}$	$\cancel{0}$
5	6	7	2	2
2	4	0	7	8

Example 9 $70300 - 59722$

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again

We repeat the process:
borrow (add) a 10
steal (subtract) a 1 again
steal (subtract) a 1

Method 1

$$
\begin{array}{rccccc}
 & & & 12 & & \\
 & 6 & 9 & \cancel{2} & 9 & 10 \\
 & \cancel{7} & \cancel{0} & \cancel{3} & \cancel{0} & \cancel{0} \\
- & 5 & 9 & 7 & 2 & 2 \\
\hline
 & 1 & 0 & 5 & 7 & 8 \\
\hline
\end{array}
$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 30

$$
\begin{array}{rccccc}
 & & & 12 & & \\
 & 6 & 9 & \cancel{2} & 9 & 10 \\
 & \cancel{7} & \cancel{0} & \cancel{3} & \cancel{0} & \cancel{0} \\
- & 5 & 9 & 7 & 2 & 2 \\
\hline
 & 1 & 0 & 5 & 7 & 8 \\
\hline
\end{array}
$$

Example 10 $70005 - 54567$

Method 1

$$
\begin{array}{rccccc}
 & 6 & 9 & 9 & 9 & 15 \\
 & \cancel{7} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{5} \\
- & 5 & 4 & 5 & 6 & 7 \\
\hline
 & 1 & 5 & 4 & 3 & 8 \\
\hline
\end{array}
$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 7000

$$
\begin{array}{rccccc}
 & 6 & 9 & 9 & 9 & 15 \\
 & \cancel{7} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{5} \\
- & 5 & 4 & 5 & 6 & 7 \\
\hline
 & 1 & 5 & 4 & 3 & 8 \\
\hline
\end{array}
$$

EASY subtraction method without having to borrow

This involves knowing negative numbers and place value!

$85 - 37$

Step 2:
Do this vertical calculation

Step 1:
Do this vertical calculation

$$
\begin{array}{r}
85 \\
-\ 37 \\
\hline
5\ \text{-}2 \\
\hline
\end{array}
$$

tens place so represents 50

ones place so represents 2

$50 - 2 = 48$

$435 - 269$

Step 3:
Do this vertical calculation

Step 2:
Do this vertical calculation

Step 1:
Do this vertical calculation

$$
\begin{array}{r}
435 \\
-\ 269 \\
\hline
2\text{-}3\text{-}4 \\
\hline
\end{array}
$$

hundreds place so represents 200

tens place so represents 30

ones place so represents 4

$200 - 30 - 4 = 166$

$\boxed{202 - 54}$

$$
\begin{array}{r}
2\ 0\ 2 \\
-\quad 5\ 4 \\
\hline
2\ {-5}\ {-2} \\
\hline
\end{array}
$$

hundreds place so represents 200

tens place so represents 50

ones place so represents 2

$$200 - 50 - 2 = 148$$

$\boxed{3400 - 2246}$

$$
\begin{array}{r}
3\ 4\ 0\ 0 \\
-\ 2\ 2\ 4\ 6 \\
\hline
1\ 2\ {-4}\ {-6} \\
\hline
\end{array}
$$

thousands place so represents 1000

hundreds place so represents 200

tens place so represents 40

ones place so represents 6

$$1000 + 200 - 40 - 6 = 1154$$

Another EASY subtraction method without having to borrow

This method involves working HORIZONTALLY and grouping!

$$\boxed{435 - 269}$$

$$435 - 269$$

$$400-200+30-60+5-9$$

$$200-30-4$$

$$166$$

$$3400 - 2246$$

3400 - 2246

3000-2000+400-200+0-40+0-6

1000+200-40-6

1154

Another Trick - Dealing With Lots Of Zeros On Top

$$5000 - 2384$$

Instead of borrowing as usual like so:

$$\begin{array}{r} \scriptstyle 4\;\;9\;\;9\;\;10 \\ \cancel{5}\,\cancel{0}\,\cancel{0}\,0 \\ -\,2\,3\,8\,4 \\ \hline \end{array}$$

Subtract 1 from each number

5000−1

Subtract 1

Subtract 1 from each number

$2384 - 1$

Subtract 1

$$\begin{array}{r} 4\,9\,9\,9 \\ -\,2\,3\,8\,3 \\ \hline 2\,6\,1\,6 \\ \hline \end{array}$$

Basic Multiplication

There are many ways to multiply which you will see in detail on the following pages:

Way 1: Area Model/Grid/Box Method – This method shows clearly what is happening and is is great for understanding, especially for those who prefer a visual understanding as it can be linked to finding the area of rectangles. It also comes in handy in other areas as it is a relatively natural method and can be used to help with expanding quadratics and multiplying polynomials.

Ways 2 and 3: Column Method – Way 3 is very widespread and more likely to be understood by parents and grandparents. It is also a nice algorithmic method that allows space to understand what is going on.

Way 4: The Lattice Method (Napier's Bones/Gelosia Method) – This is great if your main goal is just to get multiplication done, however doesn't do anything to aid understanding. The area model leads to this method. Weaker students like this method as a student who doesn't understand what multiplication is about might be able to reproduce this method and get the answer right every time. The problem is that this take time to set up and does not advance any mathematical concepts (it destroys place value).

Way 5: Criss Cross Method – This is not a very natural method, but it is quick and works for multiplying any n by n multiplication problem.

Way 6: Chinese Stick Multiplication (Line Method/Japanese Multiplication) – This method helps students to think more about what the multiplication of certain digits is providing to the product. Such as the multiplication of a ones digit and another ones digit will provide the ones digit of the product. It's one thing to know how to carry out a procedure (like long multiplication), but this is only useful when a student knows why that method works!

Note: We will look at the Criss Cross Method and Chinese Stick Multiplication method separately at the end

Way 1

Area Model/Box/Grid Method

Example 1 $\boxed{32 \times 7}$

32×7

Split/partition each of the numbers up into their place values

32= 3 tens (30) and 2 ones (2) which means 30 +2 (put on top of box)

7 = 7 ones (7) which means 7 (put on side of box)

	30	2
7	7×30 210	7×2 14

add all numbers in the boxes together: $210 + 14 = 224$

Method:

For each box we FIRST multiply the number on the top of the box with the number on the left of of the box. (see the calculations in the top left of each box which indicate this)

We then add all the numbers in the boxes together.

Way 2

Shortcut Column Method

3 2

2 1

7

Note: we write 30 and not 3
since 3 is in the tens place

1 $\boxed{2 \times 7} = 14$

2 $\boxed{30 \times 7} = 210$

$$210 + 14 = 24$$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1)

2 do this next   1 start here

$$\begin{array}{r} {}^{1}32 \\ \times \quad 7 \\ \hline \end{array}$$

3×7 7×2

add the 1 after from previous step

$3\times7+1=22$

7 x 2 =14

we carry the 1 from the 14 up like we did with addition/subtraction

$\Rightarrow$

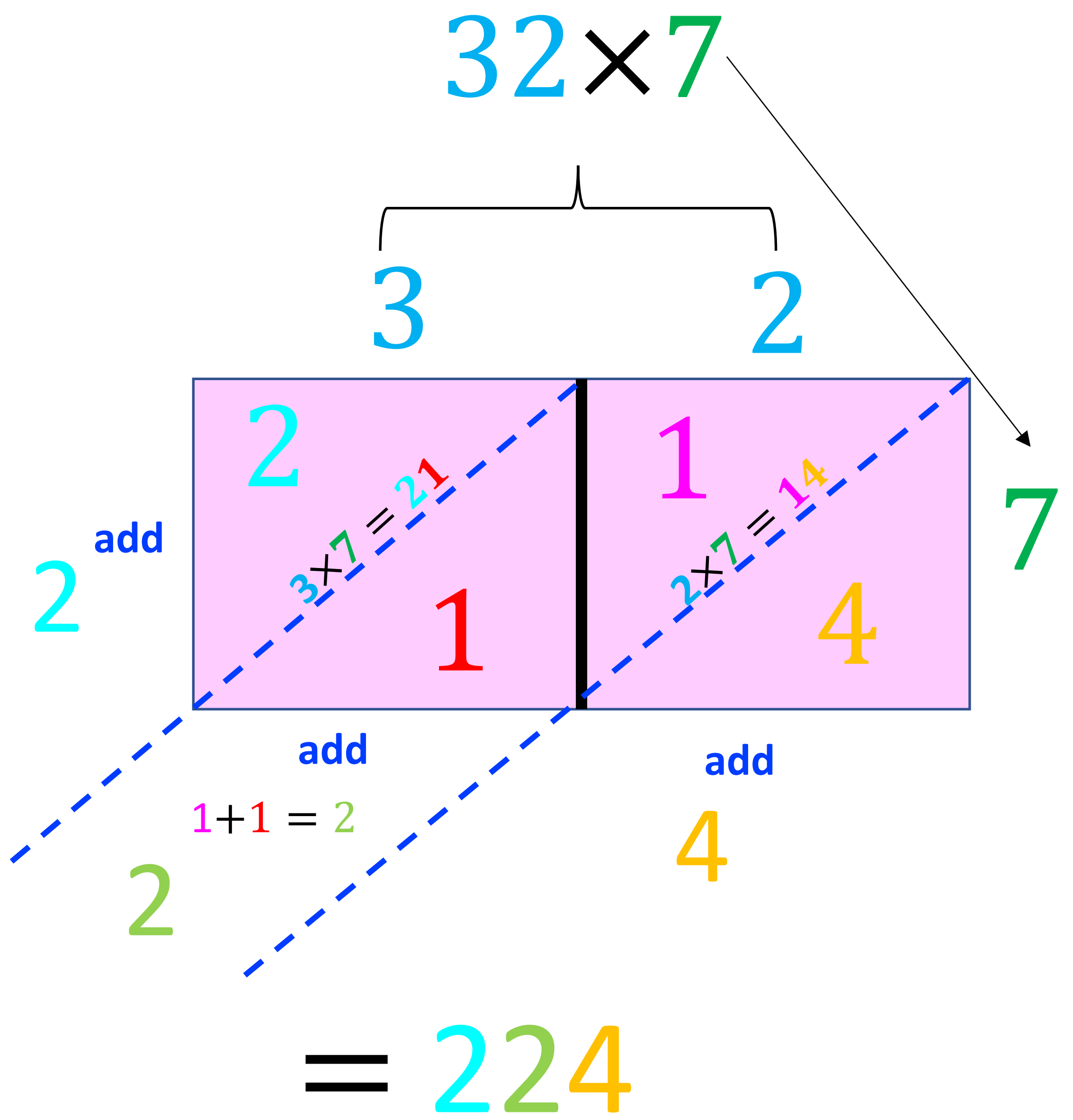
2 do this next   1 start here

$$\begin{array}{r} 32 \\ \times \quad 7 \\ \hline 224 \end{array}$$

$3\times7+1=22$

Way 4

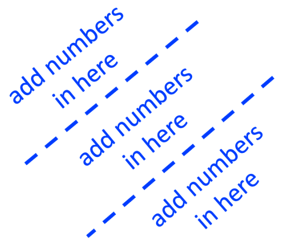
Lattice Method/Napier's Bones/Gelosia



Method:

Step 1:
For each box we FIRST multiply the numbers on the top of the box with the number to the far right of the box (7) and THEN split the digits of the number you get from multiplying (this number is shown on top of the diagonal) across the dashed diagonal that divides each box.

Step 2:
Add the numbers in each of the separate diagonal strips



(start on the right). These numbers form our answer (from left to right).

Example 2 $\boxed{43 \times 82}$

Way 1

Area Model/Box/Grid Method

Split/partition the numbers up into their place values

43= 4 tens (40) and 3 ones (3) which means 40 +3

partition here

82 = 8 tens (80) and 2 ones (2) which means 80+2

partition here

	40	3
80	80×40 3,200	80×3 240
2	2×40 80	2×3 6

Method:

For each box we FIRST multiply the number on the top of the box with the number on the left of of the box.

We then add all the numbers in the boxes together.

add all numbers in the boxes together: 3,200+ 240 + 80 +6 = 3,526

Way 2

Shortcut Column Method

$$\begin{array}{cc} 4 & 3 \\ 8 & 2 \end{array}$$

Method:

Multiply each of the colour pairs and then add the results

$\boxed{3 \times 2}$ = 6

$\boxed{3 \times 80}$ = 240

$\boxed{40 \times 2}$ = 80

$\boxed{40 \times 80}$ = 3,200

Note: we write 40 and not 4 since 4 is in the tens place

Note: we write 80 and not 8 since 8 is in the tens place

$$6 + 240 + 80 + 3{,}200 = 3{,}526$$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1/2

Step 1

do every multiplication with the pink numbers (carry if we have a two-digit number, just like with addition)

we write our answers on the top line

```
      4 3
x     8 2
---------
   2×4 2×3

---------
```

next $\Rightarrow$

Step 2

do every multiplication with the blue numbers (carry if we have a two-digit number, just like with addition)

we write our answers on the bottom line

```
      4 3
x     8 2
---------
```

done in step 1 $\longrightarrow$ 2×4 2×3

8×4 8×3 0

+2

This is 24. We need to carry the 2 over since 2 digits

always put a zero in this place

next $\Rightarrow$

Step 3

Do each calculation and then add

```
      4 3
x     8 2
---------
      8 6
+ 3 4 4 0
---------
  3 5 2 6
```

Note: This example has shown the steps, but you should be able to do just do the 3rd column once you understand the steps

Without all the colour coding this example just looks like

do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{lr} & 43 \\ \times & 82 \\ \hline & 86 \\ & \\ \hline \end{array}$$

next $\Longrightarrow$

do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{lr} & {}^{2}43 \\ \times & 82 \\ \hline & 86 \\ & 3440 \\ \hline \end{array}$$

always put a zero here

add $\Longrightarrow$

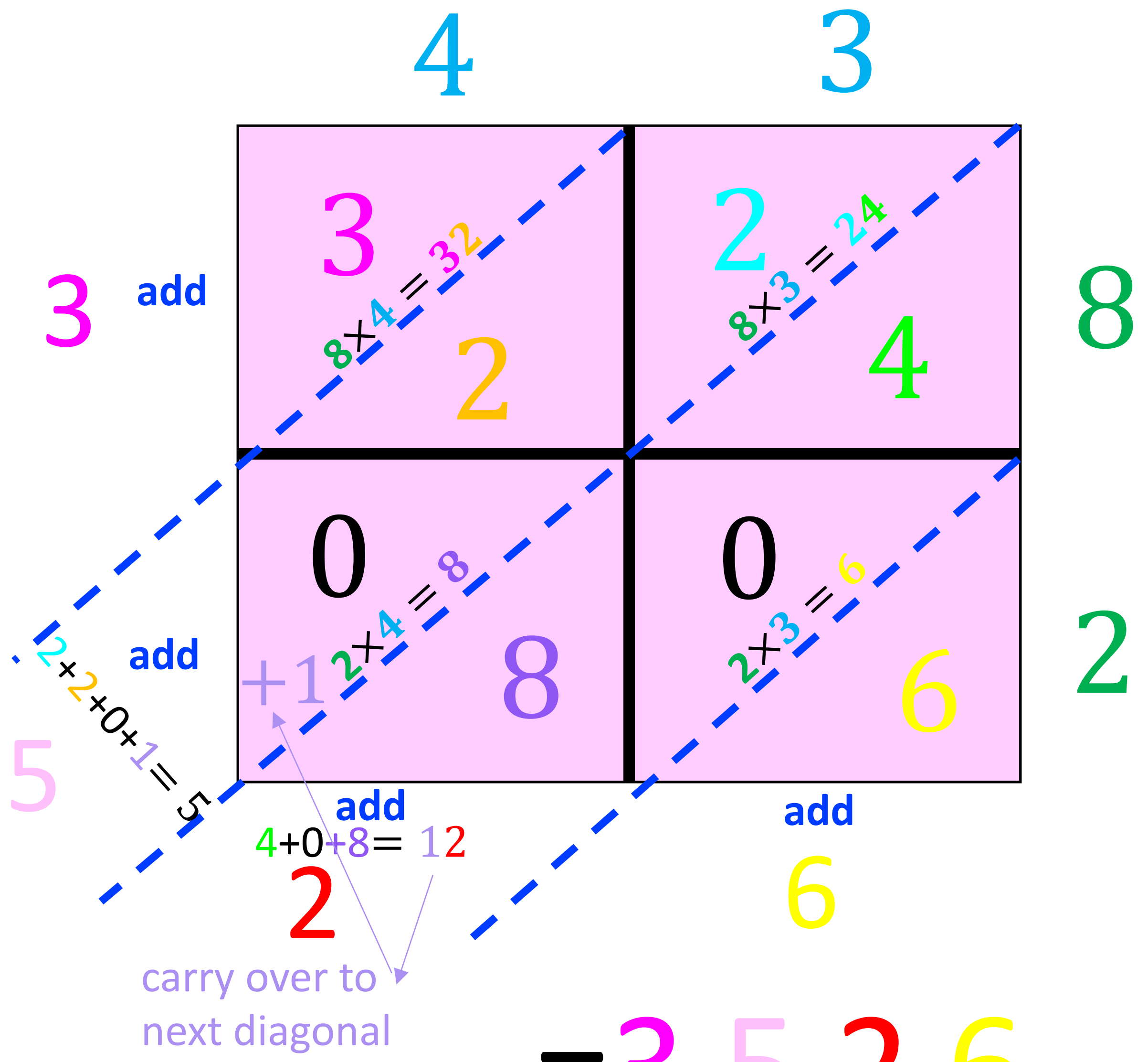
$$\begin{array}{lr} & 43 \\ \times & 82 \\ \hline & 86 \\ + & 3440 \\ \hline & 3526 \end{array}$$

Note: This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Way 4

Lattice Method

43×82

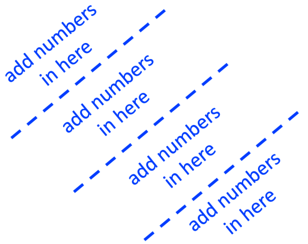


$= 3\ 5\ 2\ 6$

Method:

Step 1:
For each box we FIRST multiply the numbers on the top of the box with the numbers to the far right of the box and THEN split the digits of the number you get from multiplying (this number is shown on top of the diagonal) across the dashed diagonal that divides each box.

Step 2:
Add the numbers in each of the separate diagonal strips



(start on the right). These numbers form our answer (from left to right).

Example 3 612×24

Way 1

Area Model/Box/Grid Method

612= 6 hundreds (600), 1 tens (10) and 2 ones (2) which means 600+10 +2

partition and put here

24 = 2 tens (20) and 4 ones (4) which means 20+4

partition and put here

	600	10	2
20	20×600 12,000	20×10 200	20×2 40
4	4×600 2,400	4×10 40	4×2 8

Method

For each box we FIRST multiply the number on the top of the box with the number on the left of of the box.

We then add all the numbers in the boxes together.

$$12{,}000 + 200 + 40 + 2{,}400 + 40 + 8 = 14{,}688$$

Way 2

Shortcut Column Method

$$\begin{array}{r} 612 \\ 24 \end{array}$$

$\boxed{2 \times 4}$ = 8

$\boxed{2 \times 20}$ = 40

$\boxed{10 \times 4}$ = 40

$\boxed{10 \times 20}$ = 200

$\boxed{600 \times 4}$ = 2,400

$\boxed{600 \times 20}$ = 12,000

Method:

Multiply each of the colour pairs and then add the results

$$8+40 + 40 + 200 + 2,400 + 12,000 = 14,688$$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1/2

Step 1

do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

we write our answers on the top line

$$\begin{array}{r} 612 \\ \times \quad 24 \\ \hline 4\times6 \quad 4\times1 \quad 4\times2 \\ \hline \end{array}$$

next $\Longrightarrow$

Step 2

do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

we write our answers on the bottom line

$$\begin{array}{r} 612 \\ \times \quad 24 \\ \hline 4\times6 \quad 4\times1 \quad 4\times2 \\ 2\times6 \quad 2\times1 \quad 2\times2 \quad 0 \\ \hline \end{array}$$

always put a zero here

next $\Longrightarrow$

Do each calculation and then add

Step 3

$$\begin{array}{r} 612 \\ \times \quad 24 \\ \hline 2448 \\ +12240 \\ \hline 14688 \end{array}$$

Note: This example has shown the steps, but you should be able to do just do the 3rd column once you understand the steps

Without all the colour coding this example just looks like

do every multiplication with the pink numbers (carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} 612 \\ \times \quad 24 \\ \hline 2448 \\ \\ \hline \end{array}$$

next $\Longrightarrow$

do every multiplication with the blue numbers (carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} 612 \\ \times \quad 24 \\ \hline 2448 \\ 12240 \\ \hline \end{array}$$

always put a zero here

add $\Longrightarrow$

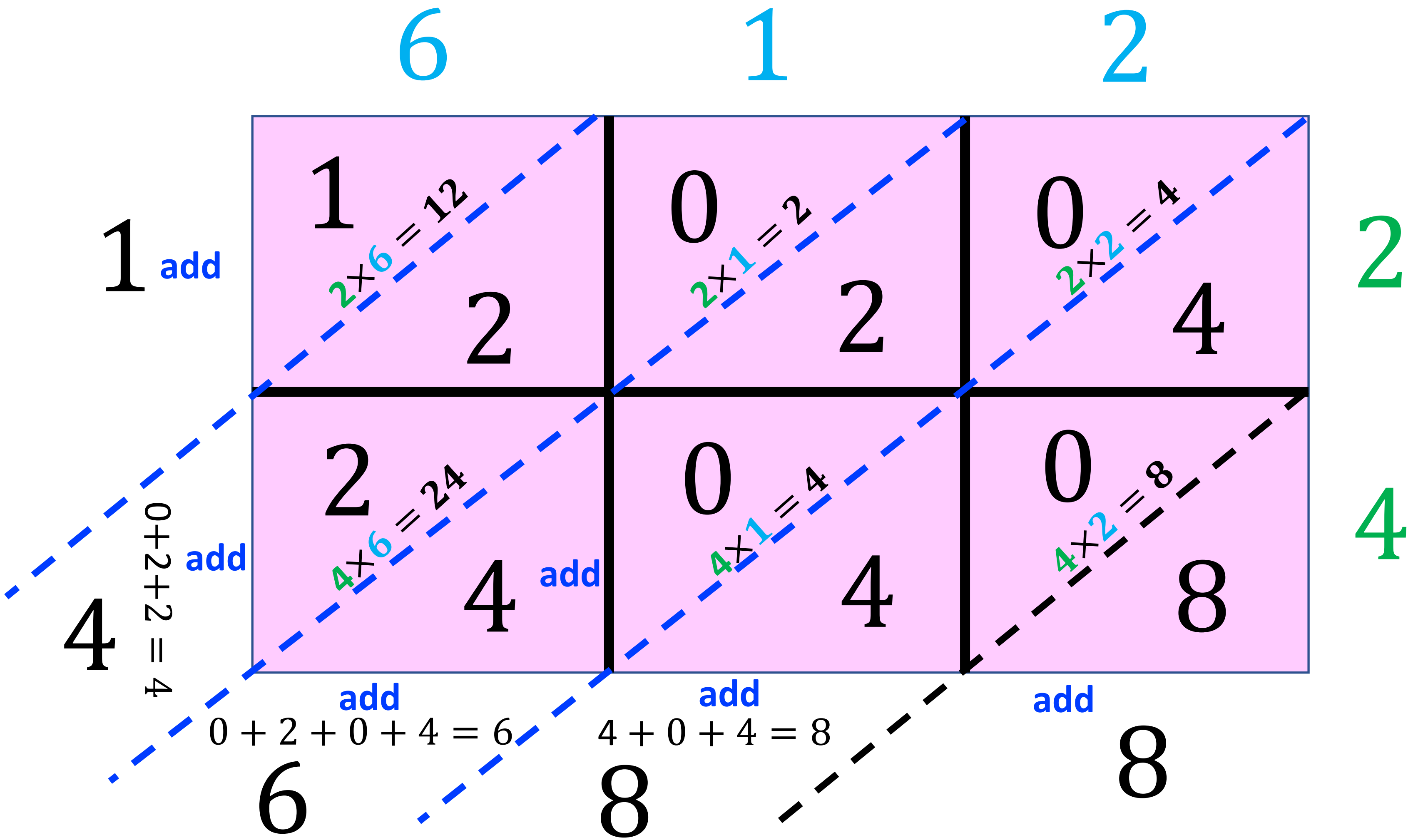
$$\begin{array}{r} 612 \\ \times \quad 24 \\ \hline 2448 \\ +12240 \\ \hline 14688 \end{array}$$

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Way 4

Lattice Method

612×24



Method:

Step 1:
For each box we FIRST multiply the numbers on the top of the box with the number on the far right of the box and THEN split the digits of the number you get from multiplying (shown on top of the diagonal) across the dashed diagonal that cuts up each box.

Step 2:
Add the numbers in each of the diagonal strips (start on the right). These numbers form our answer (from left to right).

Let's do another example, but this this time only using the most common method which is long multiplication way.
This example is the same as above, except we need to carry more.

Example 4 $\boxed{828 \times 35}$

$$\begin{array}{r} 828 \\ \times \quad 35 \\ \hline \end{array}$$

5×8 add the 1
5×2 add the 4 carry the 1 over (since this is 14(
5×8 carry the 4 over (since this is 40)

next $\Rightarrow$

$$\begin{array}{r} 828 \\ \times \quad 35 \\ \hline \end{array}$$

5×8 add the 1
5×2 add the 4 carry the 1 over
5×8 carry the 4 over

3×8
3×2 +2
3×8 carry the 2 over since this is 24
0

always put a zero here

add $\Rightarrow$

$$\begin{array}{r} 828 \\ \times \quad 35 \\ \hline 4140 \\ +24840 \\ \hline 28980 \end{array}$$

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Without all the extra colour coding this looks like:

do every multiplication with the pink numbers (carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} {}^{1}8\,{}^{4}2\,8 \\ \times \quad 3\,5 \\ \hline 4\,1\,4\,0 \\ \hline \end{array}$$

next $\Longrightarrow$

do every multiplication with the blue numbers (carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} 8\,{}^{2}2\,8 \\ \times \quad 3\,5 \\ \hline 4\,1\,4\,0 \\ 2\,4\,8\,4\,0 \\ \hline \end{array}$$

always put a zero here

add $\Longrightarrow$

$$\begin{array}{r} 8\,2\,8 \\ \times \quad 3\,5 \\ \hline 4\,1\,4\,0 \\ +\,2\,4\,8\,4\,0 \\ \hline 2\,8\,9\,8\,0 \end{array}$$

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Example 5 623×235

Way 1

Area Model/Box/Grid Method

	600	20	3
200	200×600 120,000	200×20 4,000	200×3 600
30	30×600 18,000	30×20 600	30×3 90
5	5×600 3,000	5×20 100	5×3 15

Method:

For each box we FIRST multiply the number on the top of the box with the number on the left of of the box.

We then add all the numbers in the boxes together.

$$120{,}000+4{,}000+600+18{,}000+600+90+3{,}000+100+15=146{,}405$$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1)

do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} {}^{1}6\,{}^{1}2\,3 \\ \times\ 2\,3\,5 \\ \hline 3\,1\,1\,5 \\ \hline \end{array}$$

we write our answers on the top line

next $\Longrightarrow$

do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} 6\,2\,3 \\ \times\ 2\,3\,5 \\ \hline 3\,1\,1\,5 \\ 1\,8\,6\,9\,0 \\ \hline \end{array}$$

always put a zero here

we write our answers on the second line

next $\Longrightarrow$

do every multiplication with the purple numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} 6\,2\,3 \\ \times\ 2\,3\,5 \\ \hline 3\,1\,1\,5 \\ 1\,8\,6\,9\,0 \\ +\,1\,2\,4\,6\,0\,0 \\ \hline 1\,4\,6\,4\,0\,5 \end{array}$$

WE always put two zeros here

we write our answers on the third line

Way 4

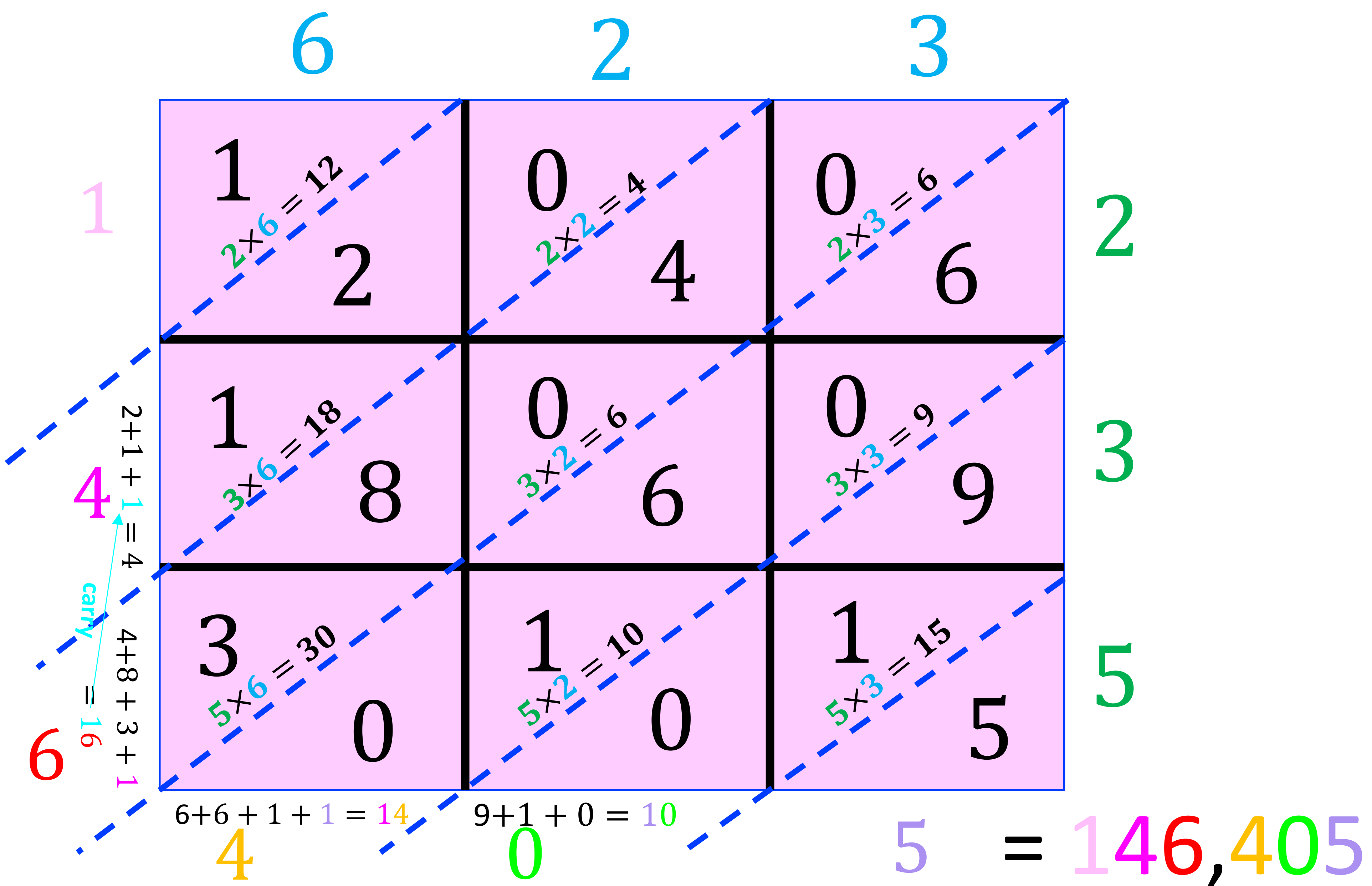
Lattice Method

623×235

Method:

Step 1:
For each box we FIRST multiply the numbers on the top of the box with the number on the far right of the box and THEN split the digits of the number you get from multiplying (shown on top of the diagonal) across the dashed diagonal that cuts up each box.

Step 2:
Add the numbers in each of the diagonal strips (start on the right). These numbers form our answer (from left to right).



Let’s now look at ways 5 and 6

Criss Cross Method

and

Chinese Stick Multiplication

Way 5

Criss Cross Method

23×12

$$
\begin{array}{r}
23 \\
\times \quad 12 \\
\hline
\end{array}
$$

③ **do this last**

$2 \times 1 = 2$

② **do this next**

$2 \times 2 = 4$

$1 \times 3 = 3$

① **start here**

$3 \times 2 = 6$

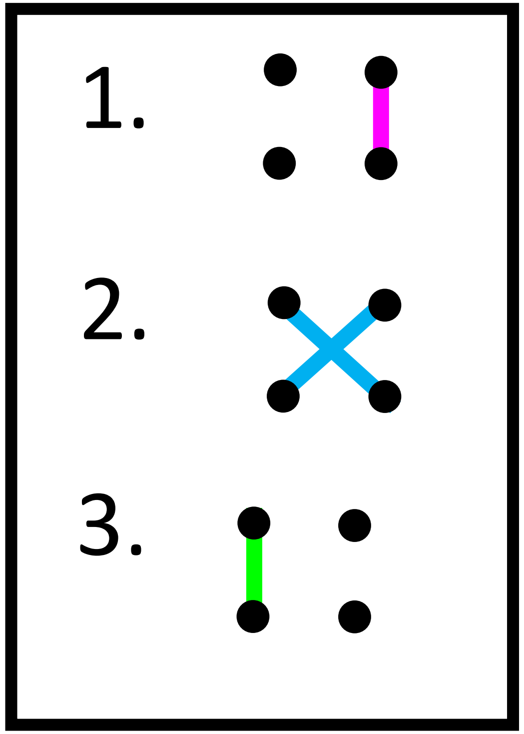
Add these numbers

$4+3=7$

$$= 276$$

Method:

We multiply each of these combinations



Way 5

Criss Cross Method

17×12

$$\begin{array}{r} 17 \\ \times\ 12 \\ \hline \end{array}$$

③ **do this last**

$1 \times 1 = 1$

Add

$1+1=2$

② **do this next**

$1 \times 2 = 2$

$1 \times 7 = 7$

Add these numbers

$2+7+1=10$

we carry the 1 to the next sum

① **start here**

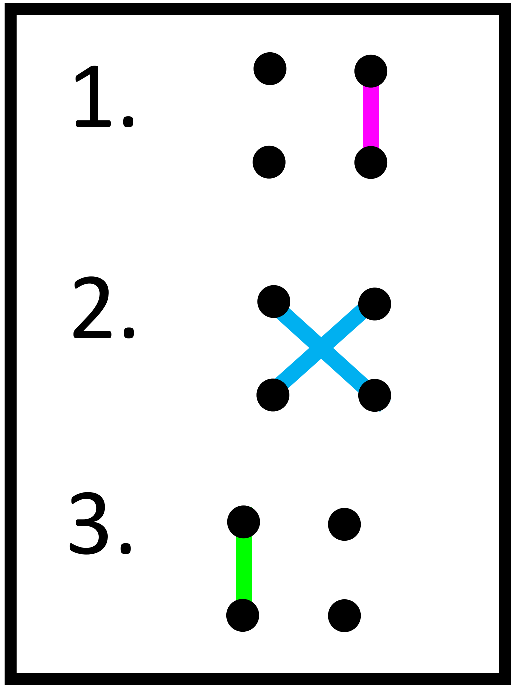
$7 \times 2 = 14$

we carry the 1 to the next sum

$= 204$

Method:

We multiply each of these combinations



Way 5

Criss Cross Method

78×25

$$\begin{array}{r} 78 \\ \times \quad 25 \\ \hline \end{array}$$

③ **do this last**

$7\times2 = 14$

Add

14+5=19

② **do this next**

$7\times5 = 35$

$2\times8 = 16$

Add these numbers

35+16+4=55

we carry the 5 to the next sum

① **start here**

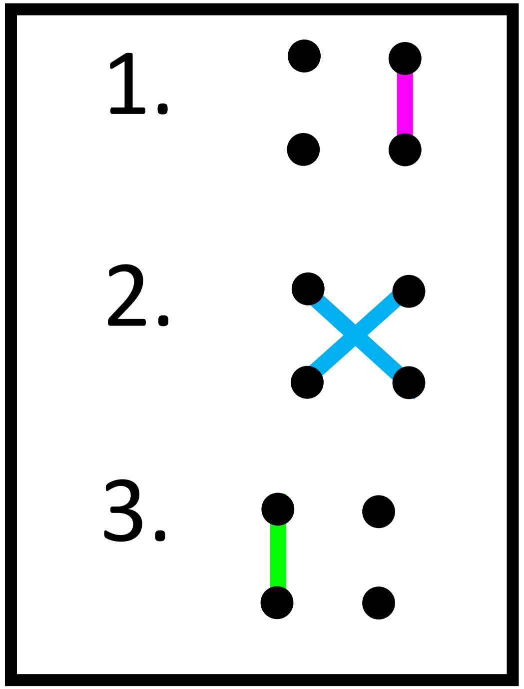
$8\times5 = 40$

we carry the 4 to the next sum

$$= 1,950$$

Method:

We multiply each of these combinations



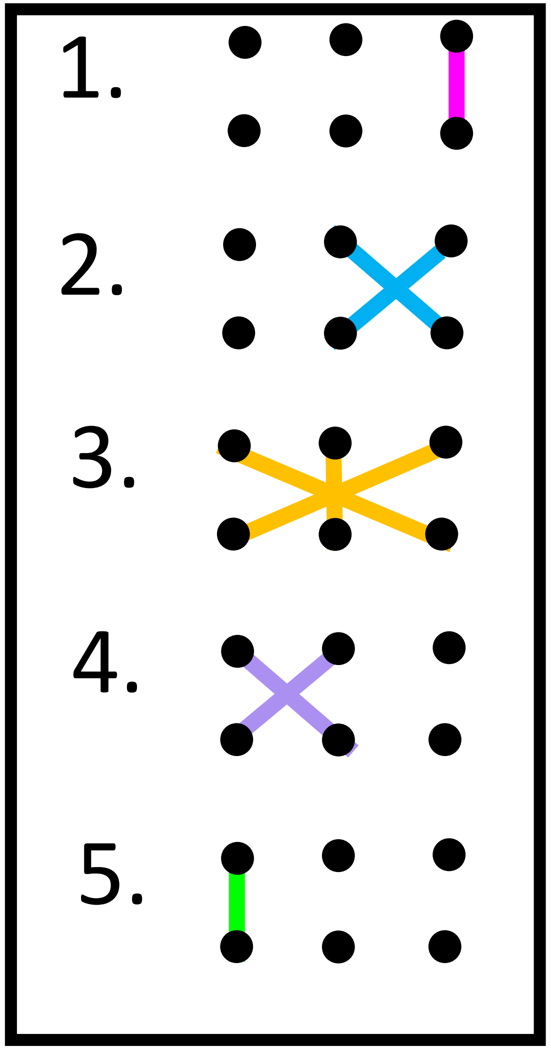
Way 5

Criss Cross Method

123×231

Method:

We multiply each of these combinations



$$\begin{array}{r} 1\,2\,3 \\ \times\ 2\,3\,1 \\ \hline \end{array}$$

① **start here**

$3\times1 = 3$

② **do this next**

$2\times1 = 2$

$3\times3 = 9$

Add these numbers

$2+9=11$

we carry the 1 to the next sum

③ **do this third**

$2\times3 = 6$

$1\times1 = 1$

$2\times3 = 6$

Add these numbers

$6+1+6+1=14$

we carry the 1 to the next sum

④ **do this fourth**

$1\times3 = 3$

$2\times2 = 4$

Add these numbers

$3+4+1= 8$

⑤ **do this last**

$1\times2 = 2$

$$= 28{,}413$$

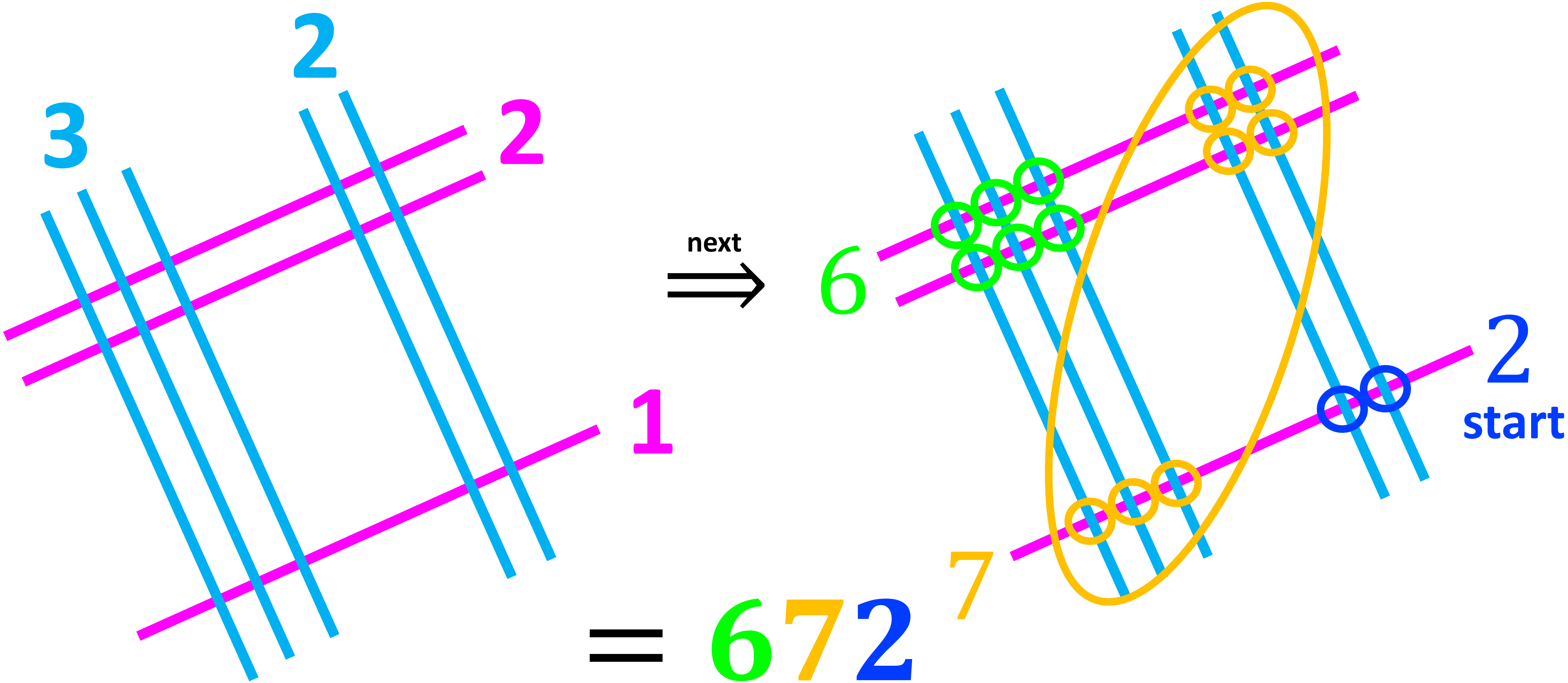
Way 6

Chinese Stick Multiplication

21×32

21×32

count the intersections for each colour group

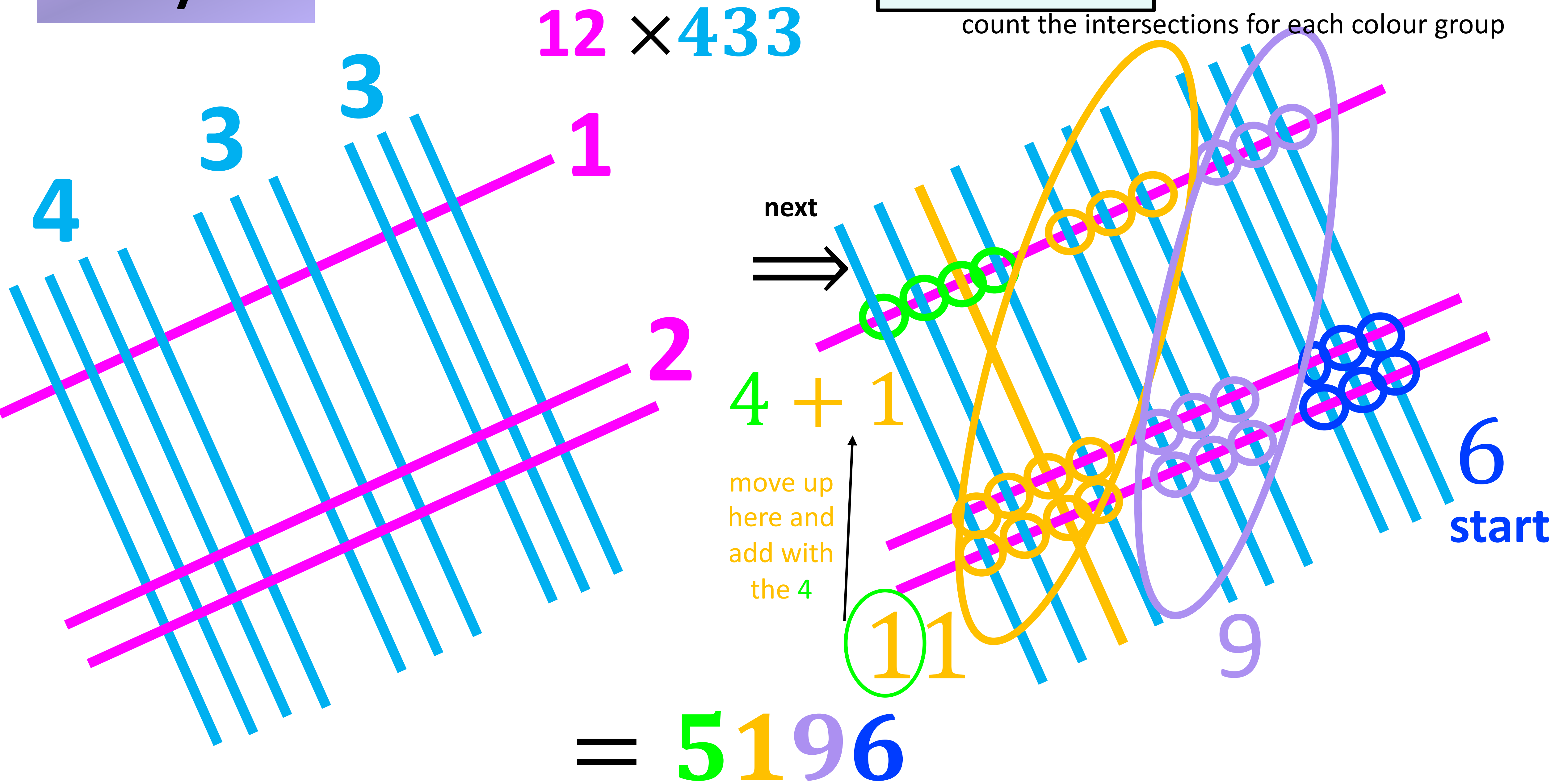


$= 672$

Way 6

Chinese Stick Multiplication

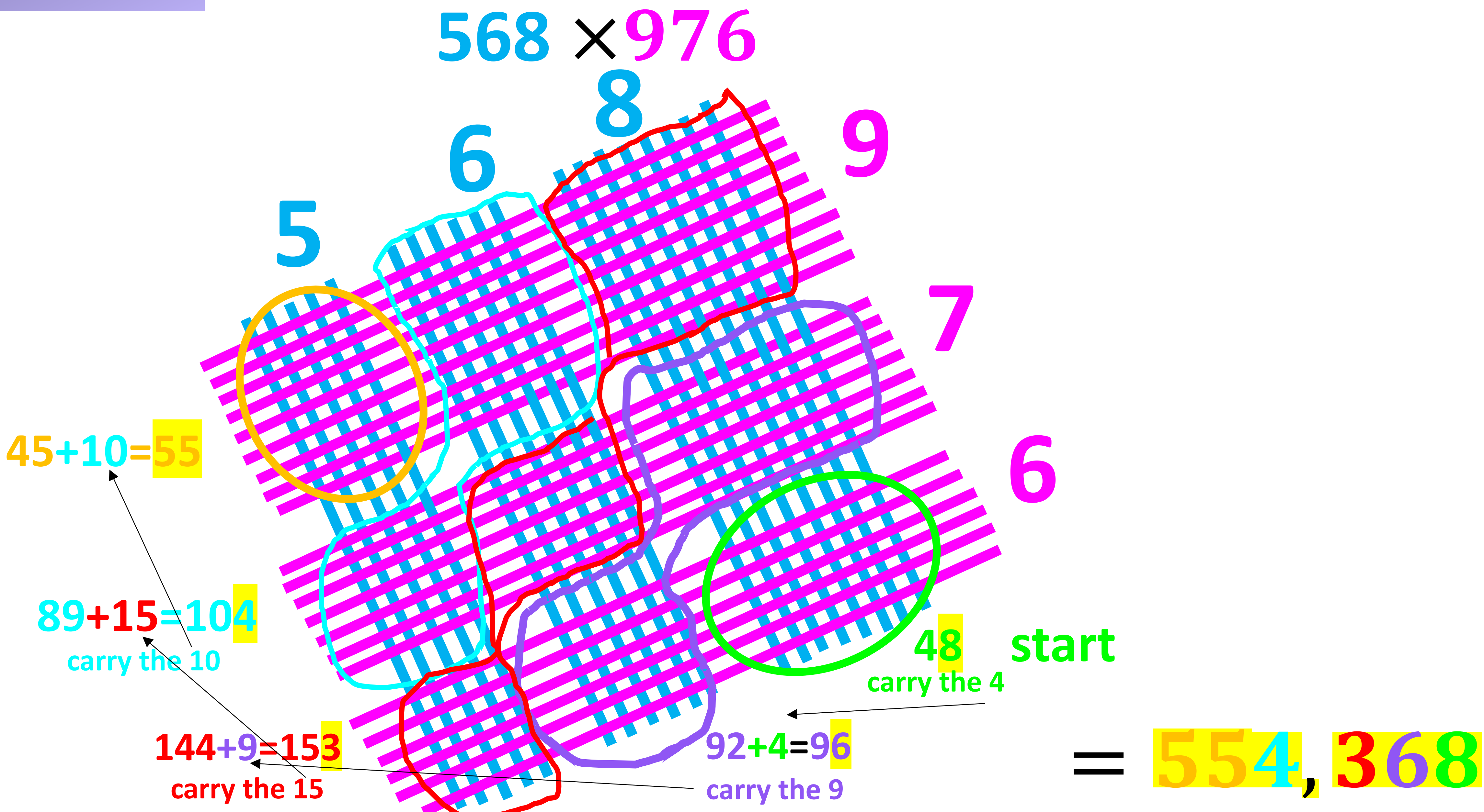
433 ×12



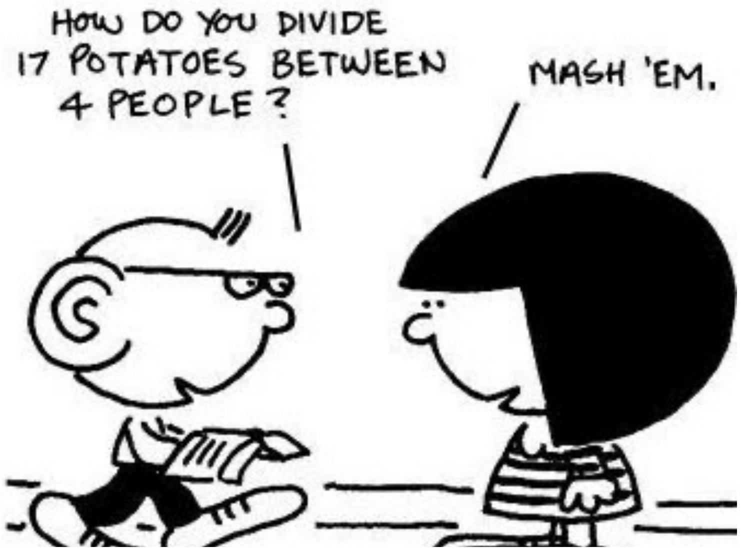
Way 6

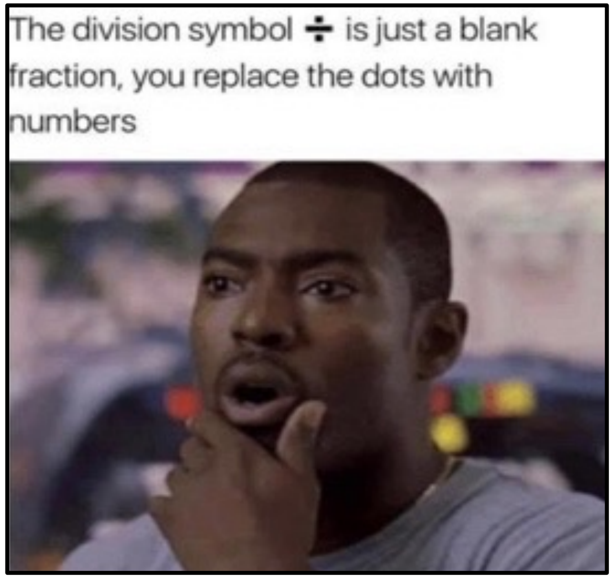
Chinese Stick Multiplication

568 ×976



Basic Division





$a \div b$ means the same as $\frac{a}{b}$ which is the same as $b\,\overline{)a}$

Notice that the numerator goes underneath the division sign: $\frac{a}{b} = \boldsymbol{b\,\overline{)a}}$

$5472 \div 3$

<u>**We now work left to right**</u>

Step 1: How many times does the number fit into each digit (each colour)

Step 2: Do the calculation to see what the result is

Step 3: Carry the remainder

$$\begin{array}{r} 1\;8\;2\;4 \\ 3\,\overline{)\,5\;{}^{2}4\;7\;{}^{1}2} \end{array}$$

How many times does 3 fit into 5?
1 time which gives 3 hence has a remainder of 2 (since 5-3=2)

How many times does 3 fit into 24?
8 times which gives 24 hence no remainder (since 24-24=0)

How many times does 3 fit into 7?
2 times which gives 6 hence a remainder of 1 (since 7-6=1)

How many times does 3 fit into 12?
4 times which gives 12 hence no remainder (since 12-12=0)

$2274 \div 6$

We now work left to right

Step 1: How many times does the number fit into each digit (each colour)

Step 2: Do the calculation to see what the result is

Step 3: Carry the remainder

$$\begin{array}{r} 0\;\;3\;\;7\;\;9 \\ 6\,\overline{)\,2\;{}^{2}2\;{}^{4}7\;{}^{5}4} \end{array}$$

How many times does 6 fit into 2?
0 times which gives 0 hence has a remainder of 2 (since 2-0=2)

How many times does 6 fit into 22?
3 times which gives 18 hence a remainder of 4 (since 22-18=4)

How many times does 6 fit into 47?
7 times which gives 42 hence a remainder of 5 (since 47-42=5)

How many times does 6 fit into 54?
9 times which gives 54 hence no remainder (since 54-54=0)

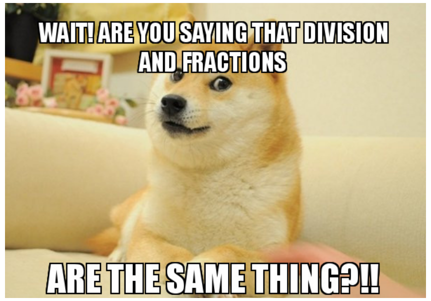
What happens if the numbers are bigger?

$$2784 \div 32$$

Option 1

We make the numbers smaller and more manageable (if possible). How we we do this:

$a \div b$ means the same thing as $\frac{a}{b}$ so we are just simplifying a fraction first and then dividing



$$2784 \div 32 = \frac{2784}{32} = \frac{1392}{16} = \frac{696}{8} = \frac{348}{4} = \frac{174}{2}$$

$$\begin{array}{r} 0\ 8\ 7 \\ 2\overline{)\,1\ {}^{1}7\ {}^{1}4} \end{array}$$

87

Option 2

Divide as normal

$$\begin{array}{r} 0\ \ 0\ \ 8\ \ 7 \\ 32\overline{)\,2\ {}^{2}7\ {}^{27}8\ {}^{22}4} \end{array}$$

It is harder to see how many times 32 fits into 278, but it is still doable

What happens if the number doesn’t fit in exactly?

$6281 \div 8$

Option 1

Divide as usual until you reach the end of the number. We write the remainder at the end

$$\begin{array}{r} 0\;7\;8\;5 \\ 8\overline{)6\;{}^{6}2\;{}^{6}8\;{}^{4}1} \end{array}$$ remainder 1

785 r 1

Option 2

We put a decimal at the end and carry on by putting zeros for as long as we need (we stop either when the number stops or when we reach our desired accuracy)

$$\begin{array}{r} 0\;7\;8\;5.1\;2\;5 \\ 8\overline{)6\;{}^{6}2\;{}^{6}8\;{}^{4}1.{}^{1}0\;{}^{2}0\;{}^{4}0} \end{array}$$

785.125