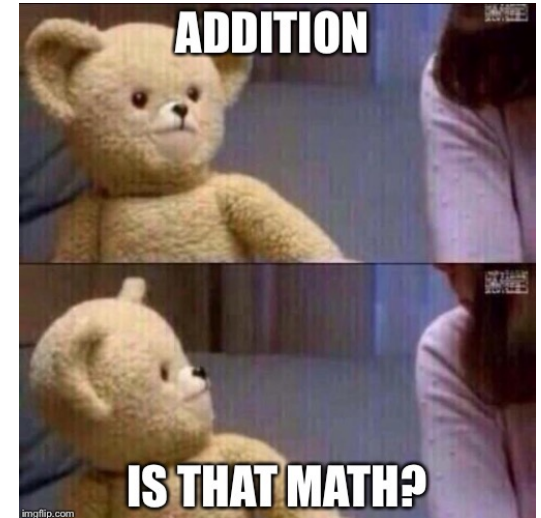


Basic Addition



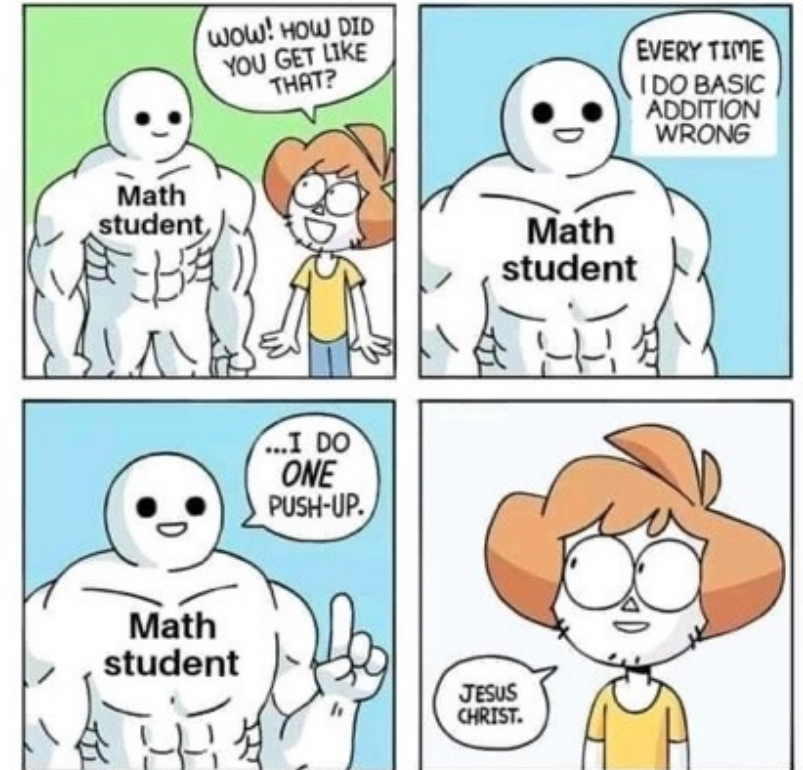
ME AS A CHILD:

7689
8905
+ 8765
<hr/>

A photograph of a young girl sitting at a desk, counting on her fingers. She has a focused expression.

ME AS AN ADULT:

10 + 5

A photograph of a Casio calculator. The display shows the equation $10 + 5 = 15$.

Example 1

$$15 + 24$$

Method

Step 1: Work on the column furthest to the right (add the digits)

Step 2: Work on next column to the left (add the digits)

If we had a bigger number we keep going from right to left column by column until we run out of columns



step ② step ①
do this next start here

$$\begin{array}{r}
 \begin{array}{cc}
 \downarrow & \downarrow \\
 1 & 5 \\
 2 & 4 \\
 \hline
 3 & 9
 \end{array}
 \end{array}$$

Note: It doesn't matter whether we put 15 or 24 on the top since adding in any order gives the same result

What happens if the digits of one of the columns add up to more than 9 i.e. if any of our column additions give a two-digit number? We will see how to deal with this on the next page.

Example 2 $16 + 38$

step step

② ①

$$\begin{array}{r} 16 \\ + 28 \\ \hline \end{array}$$

step

③

Instead, we must "carry" the 1 to the next calculation



we bring the first digit up to join the next calculation

1

step step

⑤ ④

$$\begin{array}{r} 16 \\ + 38 \\ \hline \end{array}$$

step

⑥

This gives



step

⑧

step

⑦

$$\begin{array}{r} 16 \\ + 38 \\ \hline 54 \end{array}$$

14



It is not ok to write a two-digit number here

Note: This example has shown the stages and steps to explain, but you should be able to do the final column on the right straight away. This will be shown in the next examples with 1 stage only.

Further Examples

$$63 + 82$$

$$\begin{array}{r} 63 \\ + 82 \\ \hline 145 \end{array}$$

Note It is ok here to write two-digits here since it is our final calculation.

$$426 + 395$$

$$\begin{array}{r} \overset{1}{4} \overset{1}{2} 6 \\ + 395 \\ \hline 821 \end{array}$$

12 bring the 1 up 11 bring the 1 up

$$435 + 977$$

$$\begin{array}{r} \overset{1}{4} \overset{1}{3} 5 \\ + 977 \\ \hline 1412 \end{array}$$

14. We don't need to bring the 1 up since we are done 11 bring the 1 up 12 bring the 1 up

What happens if some of the digits are missing? Fill in any gaps with zeros and add as normal

$$213 + 92$$

$$\begin{array}{r} \overset{1}{2} 1 3 \\ + 0 9 2 \\ \hline 3 0 5 \end{array}$$

Basic Subtraction

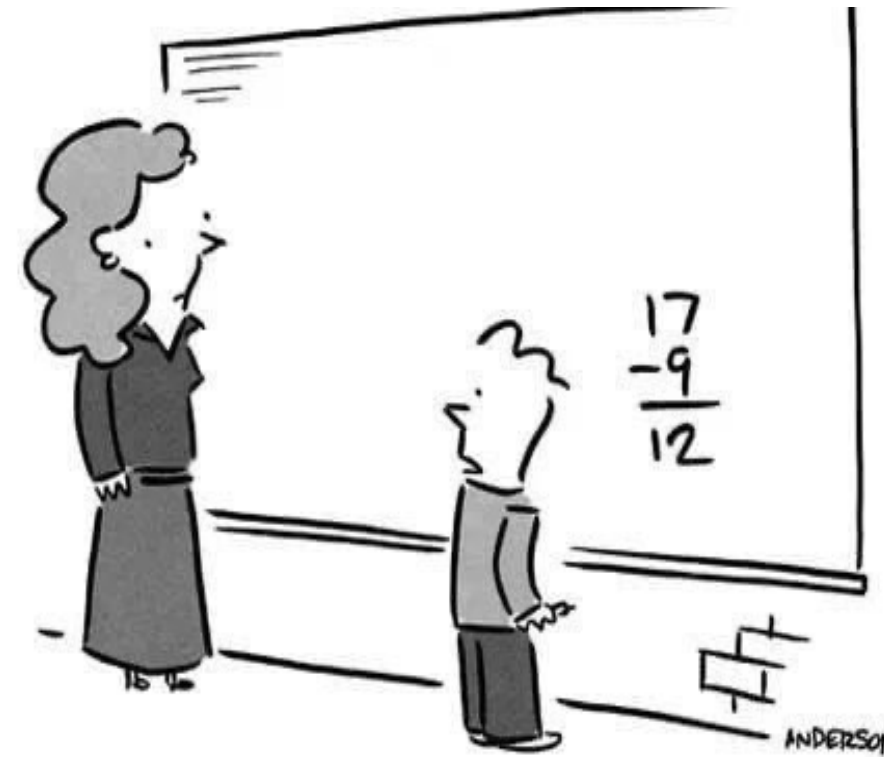
How good you are in mathematics ?

Me :



Scientist: students need 8-10 hours of sleep a day

School:



"I know it's wrong, I'm just waiting for the autocorrect."

Example 1

$$85 - 24$$

Method

Step 1: Work on the column furthest to the right (subtract the digits)

Step 2: Work on next column to the left (subtract the digits)

If we had a bigger number we keep going from right to left column by column until we run out of columns



② do this next ① start here

$$\begin{array}{r}
 \textcircled{2} \quad \textcircled{1} \\
 \text{do this next} \quad \text{start here} \\
 \downarrow \quad \downarrow \\
 \begin{array}{r}
 85 \\
 - 24 \\
 \hline
 61
 \end{array}
 \end{array}$$

Note: Unlike with addition, we must put the first number on the top since subtraction in any order does not give the same result

For example:

$$4 - 2 = 2$$

but

$$2 - 4 = -2$$

What happens if the digit at the bottom is greater than the digit on the top in any column? We will see how to deal with this on the next page.

Example 2

$$85 - 37$$

Method

borrow (add) a 10 since 5 is less than 7
steal from the next column (subtract) a 1

② ①
do this next start here

$$\begin{array}{r}
 \begin{array}{cc}
 \text{7} \swarrow & \downarrow \\
 \text{8} & \text{5} \\
 \swarrow & \swarrow \\
 & \text{15}
 \end{array} \\
 - \begin{array}{cc}
 \text{3} & \text{7}
 \end{array} \\
 \hline
 \begin{array}{cc}
 \text{4} & \text{8}
 \end{array} \\
 \hline
 \end{array}$$

This is different to the last example. Why?

For **each calculation** we always need a bigger number on top. Here we do not have that for the **pink calculation** (7 is bigger than 5), so we need to **borrow** and steal. We always **borrow 10** (add 10) for the first calculation and **steal 1** (subtract 1) for the next calculation.

Example 3

$$435 - 269$$

Method

borrow (add) a 10
steal (subtract) a 1

This time we have to
repeat the process:

borrow (add) a 10
steal (subtract) a 1

$$\begin{array}{r}
 3 \quad 12 \quad 15 \\
 \cancel{4} \quad \cancel{3} \quad \cancel{5} \\
 2 \quad 6 \quad 9 \\
 - \\
 \hline
 1 \quad 6 \quad 6 \\
 \hline
 \hline
 \end{array}$$

This is harder than the last example. Why?

Since we have to borrow and steal **TWICE**:

For **each calculation** we always need a bigger number on top. Here we do not have that for the **pink calculation** **AND** the **blue calculation**, so we need to borrow and steal.

Example 4

$$202 - 54$$

This is harder than the last example since we are dealing with a **0** when we steal which is a little more confusing:

Method 1

We proceed as usual, but here we need to take 1 away from 0.

When we take away 1 from 0 we are basically taking 1 away from 10 and therefore we turn the 0 into a 9. When we make a 0 and 9, we then ALSO AUTOMATICALLY make the next number 1 less.

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again
(since we made a 0 a 9)

$$\begin{array}{r}
 1 \quad 9 \quad 12 \\
 \cancel{2} \quad \boxed{\cancel{0}} \quad 2 \\
 - \quad 5 \quad 4 \\
 \hline
 1 \quad 4 \quad 8
 \end{array}$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 20 to get 19

$$\begin{array}{r}
 1 \quad 9 \quad 12 \\
 \cancel{2} \quad \boxed{\cancel{0}} \quad 2 \\
 - \quad 5 \quad 4 \\
 \hline
 1 \quad 4 \quad 8
 \end{array}$$

Example 5

$$3400 - 2246$$

Method 1

We take away 1 from 0 we are basically taking 1 away from 10. We have to ALSO make the next number 1 less each time we change a 0 into a 9 and hence we and do it again

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again
(since we made a 0 a 9)

		3	9	10
-	3	4	0	0
	2	2	4	6
	1	1	5	4

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 40 to get 39

		3	9	10
-	3	4	0	0
	2	2	4	6
	1	1	5	4

Example 6

$$3400 - 2746$$

This is harder than the last example since we borrow and steal twice:

Method 1

We take away 1 from 0 we are basically taking 1 away from 10. We have to ALSO make the next number 1 less each time we change a 0 into a 9 and hence we and do it again

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again

We repeat the process:
borrow (add) a 10
steal (subtract) a 1 again

$$\begin{array}{r}
 13 \\
 3400 \\
 - 2746 \\
 \hline
 0654
 \end{array}$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 40

$$\begin{array}{r}
 13 \\
 3400 \\
 - 2746 \\
 \hline
 0654
 \end{array}$$

Example 7

$$39000 - 26453$$

This is harder than the last example since we have successive 0's. Remember that with 0's we keep going:

Method 1

We take away 1 from 0 we are basically taking 1 away from 10. We have to ALSO make the next number 1 less each time we change a 0 into a 9 and hence we and do it again

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again
steal (subtract) a 1 again

		8	9	9	10
3	9	0	0	0	
2	6	4	5	3	
1	2	5	4	7	

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 900

		8	9	9	10
3	9	0	0	0	
2	6	4	5	3	
1	2	5	4	7	

Example 8

$$80800 - 56722$$

Method 1

Note: This zero did not become a 9, since we were done after the 8 became a 7 and we start the process of borrowing and stealing again

Method

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again

We repeat the process:
borrow (add) a 10
steal (subtract) a 1

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 7 & 10 & 7 & 9 & 10 \\
 \cancel{8} & \cancel{0} & \cancel{8} & \cancel{0} & \cancel{0} & \\
 5 & 6 & 7 & 2 & 2 & \\
 \hline
 2 & 4 & 0 & 7 & 8 &
 \end{array} \\
 \hline
 \end{array}$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 80

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 7 & 10 & 7 & 9 & 10 \\
 8 & \cancel{0} & \cancel{8} & \cancel{0} & \cancel{0} & \\
 5 & 6 & 7 & 2 & 2 & \\
 \hline
 2 & 4 & 0 & 7 & 8 &
 \end{array} \\
 \hline
 \end{array}$$

$$70300 - 59722$$

Example 9

Method 1

Method 2

Method

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 30

borrow (add) a 10
steal (subtract) a 1
steal (subtract) a 1 again

We repeat the process:
borrow (add) a 10
steal (subtract) a 1 again
steal (subtract) a 1

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 6 & 9 & 2 & 9 & 10 \\
 & \cancel{7} & \cancel{0} & \cancel{3} & \cancel{0} & \cancel{0} \\
 - & 5 & 9 & 7 & 2 & 2 \\
 \hline
 & 1 & 0 & 5 & 7 & 8
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 6 & 9 & 2 & 9 & 10 \\
 & \cancel{7} & \cancel{0} & \cancel{3} & \cancel{0} & \cancel{0} \\
 - & 5 & 9 & 7 & 2 & 2 \\
 \hline
 & 1 & 0 & 5 & 7 & 8
 \end{array}
 \end{array}$$

Example 10

$$70005 - 54567$$

Method 1

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 6 & 9 & 9 & 9 & 15 \\
 \cancel{7} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{5} & \\
 5 & 4 & 5 & 6 & 7 & \\
 \hline
 1 & 5 & 4 & 3 & 8 &
 \end{array}
 \end{array}$$

Method 2

when stealing from a 0, combine it with the number to the left of it i.e. steal 1 from 7000

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 6 & 9 & 9 & 9 & 15 \\
 \cancel{7} & 0 & 0 & 0 & \cancel{5} & \\
 5 & 4 & 5 & 6 & 7 & \\
 \hline
 1 & 5 & 4 & 3 & 8 &
 \end{array}
 \end{array}$$

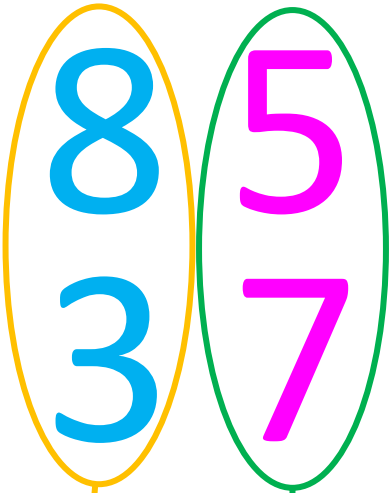
EASY subtraction
method without
having to borrow



This involves knowing negative numbers and place value!

$$85 - 37$$

Step 2:
Do this vertical
calculation



Step 1:
Do this vertical
calculation

$$\begin{array}{r} 8 \\ - 3 \\ \hline 5 \end{array} \quad \begin{array}{r} 5 \\ - 7 \\ \hline -2 \end{array}$$

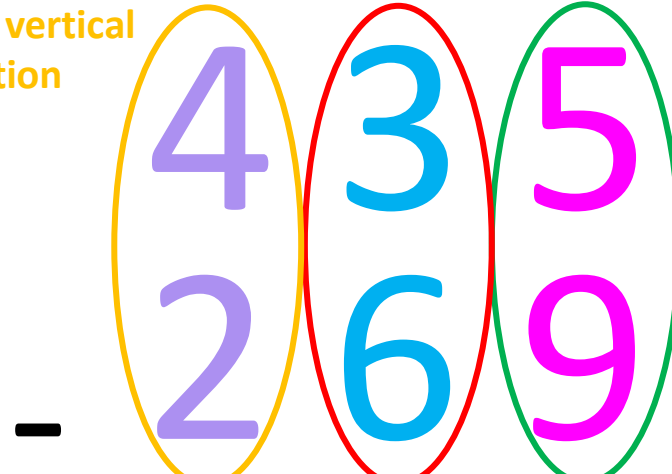
tens place so
represents 50

ones place so
represents 2

$$50 - 2 = 48$$

$$435 - 269$$

Step 3:
Do this vertical
calculation



Step 2:
Do this vertical
calculation

Step 1:
Do this vertical
calculation

$$\begin{array}{r} 4 \\ - 2 \\ \hline 2 \end{array} \quad \begin{array}{r} 3 \\ - 6 \\ \hline -3 \end{array} \quad \begin{array}{r} 5 \\ - 9 \\ \hline -4 \end{array}$$

hundreds place so
represents 200

tens place so
represents 30

ones place so
represents 4

$$200 - 30 - 4 = 166$$

$$202 - 54$$

$$\begin{array}{r} 202 \\ - 54 \\ \hline \end{array}$$

$$\underline{2} - \underline{5} - \underline{2}$$

hundreds place so
represents 200

tens place so
represents 50

ones place so
represents 2

$$200 - 50 - 2 = 148$$

$$3400 - 2246$$

$$\begin{array}{r} 3400 \\ - 2246 \\ \hline \end{array}$$

$$\underline{1} \underline{2} - \underline{4} - \underline{6}$$

thousands place so
represents 1000

hundreds place so
represents 200

tens place so
represents 40

ones place so
represents 6

$$1000 + 200 - 40 - 6 = 1154$$

Another EASY
subtraction method
without having to
borrow



This method involves working HORIZONTALLY and grouping!

$$435 - 269$$

$$435 - 269$$

$$400 - 200 + 30 - 60 + 5 - 9$$

$$200 - 30 - 4$$

$$166$$

$$3400 - 2246$$

$$3400 - 2246$$

$$3000 - 2000 + 400 - 200 + 0 - 40 + 0 - 6$$

$$1000 + 200 - 40 - 6$$

$$1154$$

Another Trick - Dealing With Zeros

$$5000 - 2384$$

Instead of borrowing as usual
like so:

	4	9	9	10
-	5	0	0	0
	2	3	8	4
<hr/>				

Subtract 1 from each number

$$5000 - 1$$



Subtract 1

Subtract 1 from each number

$$2384 - 1$$

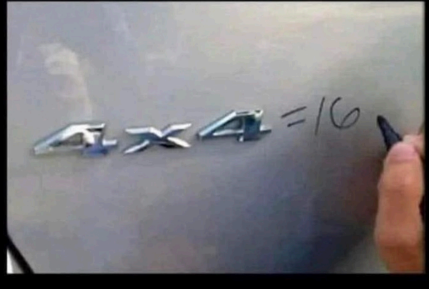


Subtract 1

	4	9	9	9
-	2	3	8	3
<hr/>				
	2	6	1	6
<hr/>				

Basic Multiplication

This poor guy was driving around with this unsolved problem on his truck, had to fix it for him...



Japanese family found calculator in their son's room



There are many ways to multiply:

Way 1: Area Model/Grid/Box Method – This method shows clearly what is happening and is great for understanding, especially for those who prefer a visual understanding as it can be linked to finding the area of rectangles. It also comes in handy in other areas as it is a relatively natural method and can be used to help with expanding quadratics and multiplying polynomials.

Ways 2 and 3: Column Method – Way 3 is very widespread and more likely to be understood by parents and grandparents. It is also a nice algorithmic method that allows space to understand what is going on.

Way 4: The Lattice Method (Napier's Bones/Gelosia Method) – This is great if your main goal is just to get multiplication done, however doesn't do anything to aid understanding. The area model leads to this method. Weaker students like this method as a student who doesn't understand what multiplication is about might be able to reproduce this method and get the answer right every time. The problem is that this takes time to set up and does not advance any mathematical concepts (it destroys place value).

Way 5: Criss Cross Method – This is not a very natural method, but it is quick and works for multiplying any n by n multiplication problem.

Way 6: Chinese Stick Multiplication (Line Method/Japanese Multiplication) – This method helps students to think more about what the multiplication of certain digits is providing to the product. Such as the multiplication of a ones digit and another ones digit will provide the ones digit of the product. It's one thing to know how to carry out a procedure (like long multiplication), but this is only useful when a student knows why that method works!

Note: We will look at the Criss Cross Method and Chinese Stick Multiplication method separately at the end

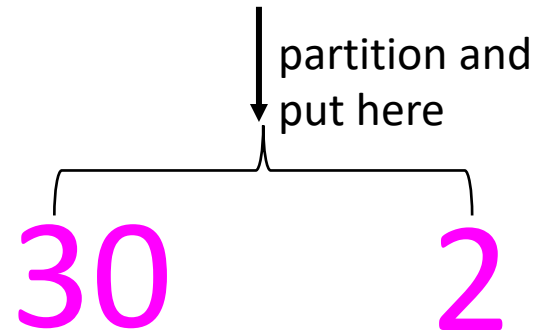
Example 1

$$32 \times 7$$

Way 1

Area Model/Box/Grid Method

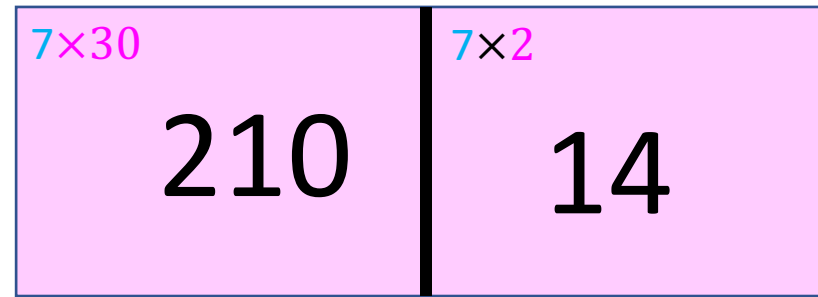
Split/partition the numbers up into their place values

 $32 = 3 \text{ tens (30) and } 2 \text{ ones (2) which means } 30 + 2$


$7 = 7 \text{ ones (7)}$
which means 7

put
here

→ 7



add all numbers in the boxes together: $210 + 14 = 224$

Method:

For each box we FIRST multiply **the number on the top of the box** with **the number on the left of the box**.

We then add all the numbers in the boxes together.

Way 2

$$\begin{array}{r} 32 \\ \times 7 \\ \hline \end{array}$$

Note: we write 30 and not 3
since 3 is in the tens place

$$2 \times 7 = 14$$

$$30 \times 7 = 210$$

$$210 + 14 = 224$$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1)

© mymathscloud

② do this next ① start here

X

3×7 7×2
add the 1 after from previous step $7 \times 2 = 14$ we carry the 1 from the 14 up like we did with addition/subtraction

$3 \times 7 + 1 = 22$

② do this next ① start here

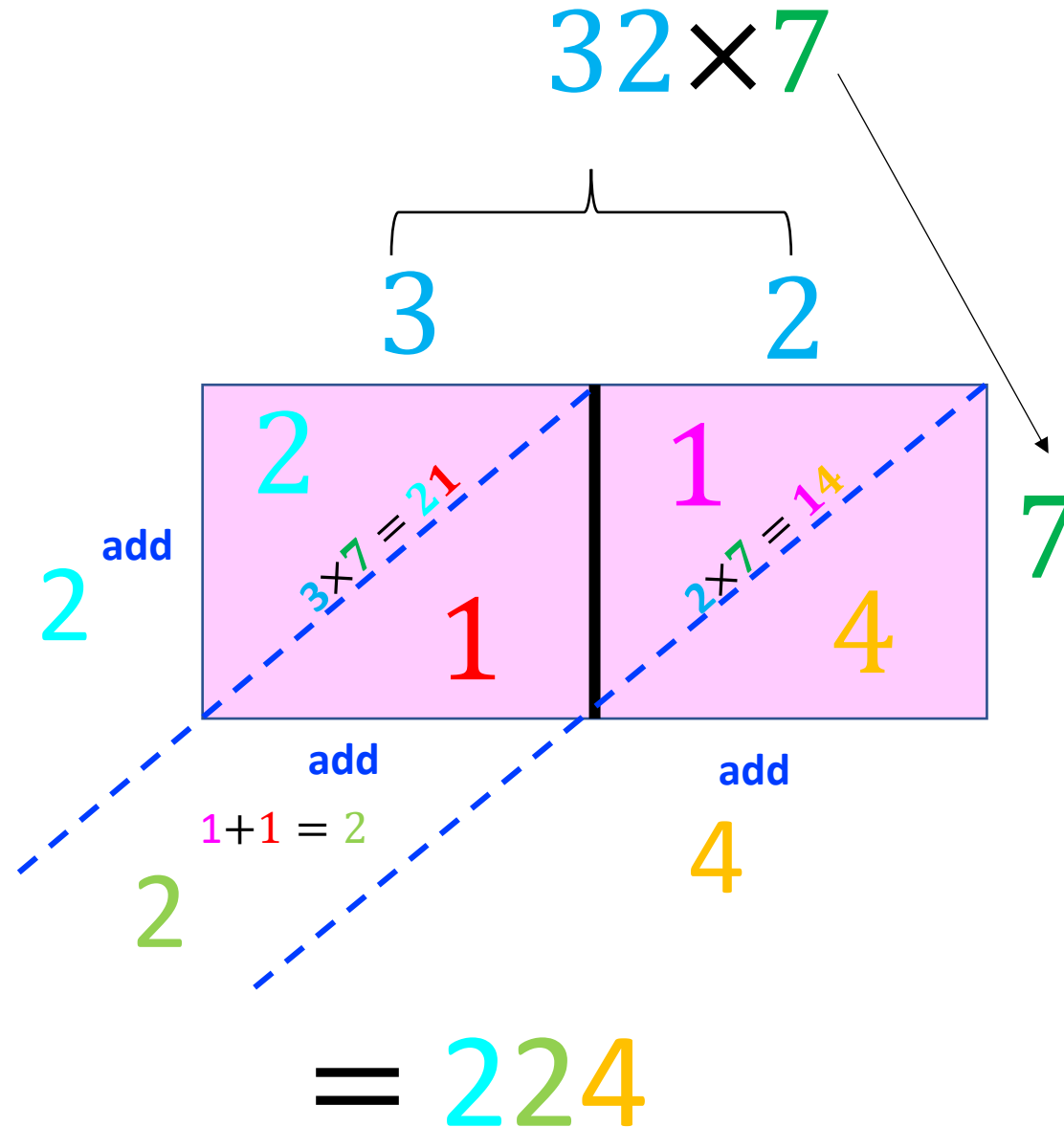
X

2 2 4

$3 \times 7 + 1 = 22$

Way 4

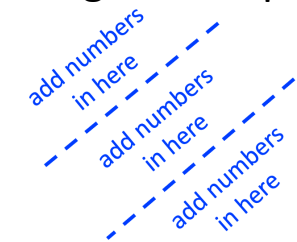
Lattice Method/Napier's Bones/Gelosia

**Method:**Step 1:

For each box we FIRST multiply the numbers on the top of the box with the number to the far right of the box (7) and THEN split the digits of the number you get from multiplying (this number is shown on top of the diagonal) across the dashed diagonal that divides each box.

Step 2:

Add the numbers in each of the separate diagonal strips



(start on the right). These numbers form our answer (from left to right).

Example 2

$$43 \times 82$$

Way 1

Area Model/Box/Grid Method

Split/partition the numbers up into their place values

 $43 = 4 \text{ tens (40) and 3 ones (3) which means } 40 + 3$

partition here

40

3

82 = 8 tens (80)
and 2 ones (2)
which means
80+2

partition here

80

2

80×40 3,200	80×3 240
2×40 80	2×3 6

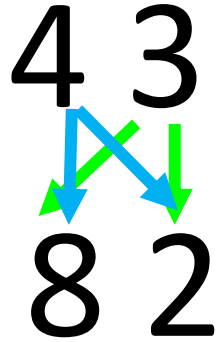
Method:

For each box we FIRST multiply **the number on the top of the box** with **the number on the left of the box**.

We then add all the numbers in the boxes together.

add all numbers in the boxes together: $3,200 + 240 + 80 + 6 = 3,526$

Way 2



$$3 \times 2 = 6$$

$$3 \times 80 = 240$$

$$40 \times 2 = 80$$

$$40 \times 80 = 3,200$$

Note: we write 40 and not 4
since 4 is in the tens place

Note: we write 80 and not 8
since 8 is in the tens place

Method:

Multiply each of the
colour pairs and then add
the results

$$6 + 240 + 80 + 3,200 = 3,526$$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1/2)

© mymathscloud

Step 1

do every multiplication with the pink numbers (carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} \text{X} \quad 43 \\ \hline 82 \\ \hline \end{array}$$

2×4 2×3

we write our answers on the top line

next \Rightarrow

Step 2

do every multiplication with the blue numbers (carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} \text{X} \quad 43 \\ \hline 82 \\ \hline \end{array}$$

done in step 1 \rightarrow 2×4 2×3

8×4 8×3 0

$+2$

This is 24. We need to carry the 2 over since 2 digits

we write our answers on the bottom line

next \Rightarrow

Do each calculation and then add

$$\begin{array}{r} \text{X} \quad 43 \\ \hline 82 \\ \hline \end{array}$$

8 6

$+$ 34 40

3526

always put a zero in this place

Note: This example has shown the steps, but you should be able to do just do the 3rd column once you understand the steps

Without all the colour coding this looks like

© mymathscloud

do every multiplication with
the pink numbers
(carry if we have a two-digit
number, just like with addition)

$$\begin{array}{r} 43 \\ \times 82 \\ \hline 86 \\ \hline \end{array}$$

next \Rightarrow

do every multiplication with
the blue numbers
(carry if we have a two-digit
number, just like with addition)

$$\begin{array}{r} 43 \\ \times 82 \\ \hline 86 \\ 3440 \\ \hline \end{array}$$

add \Rightarrow

$$\begin{array}{r} 43 \\ \times 82 \\ \hline 86 \\ + 3440 \\ \hline 3526 \end{array}$$

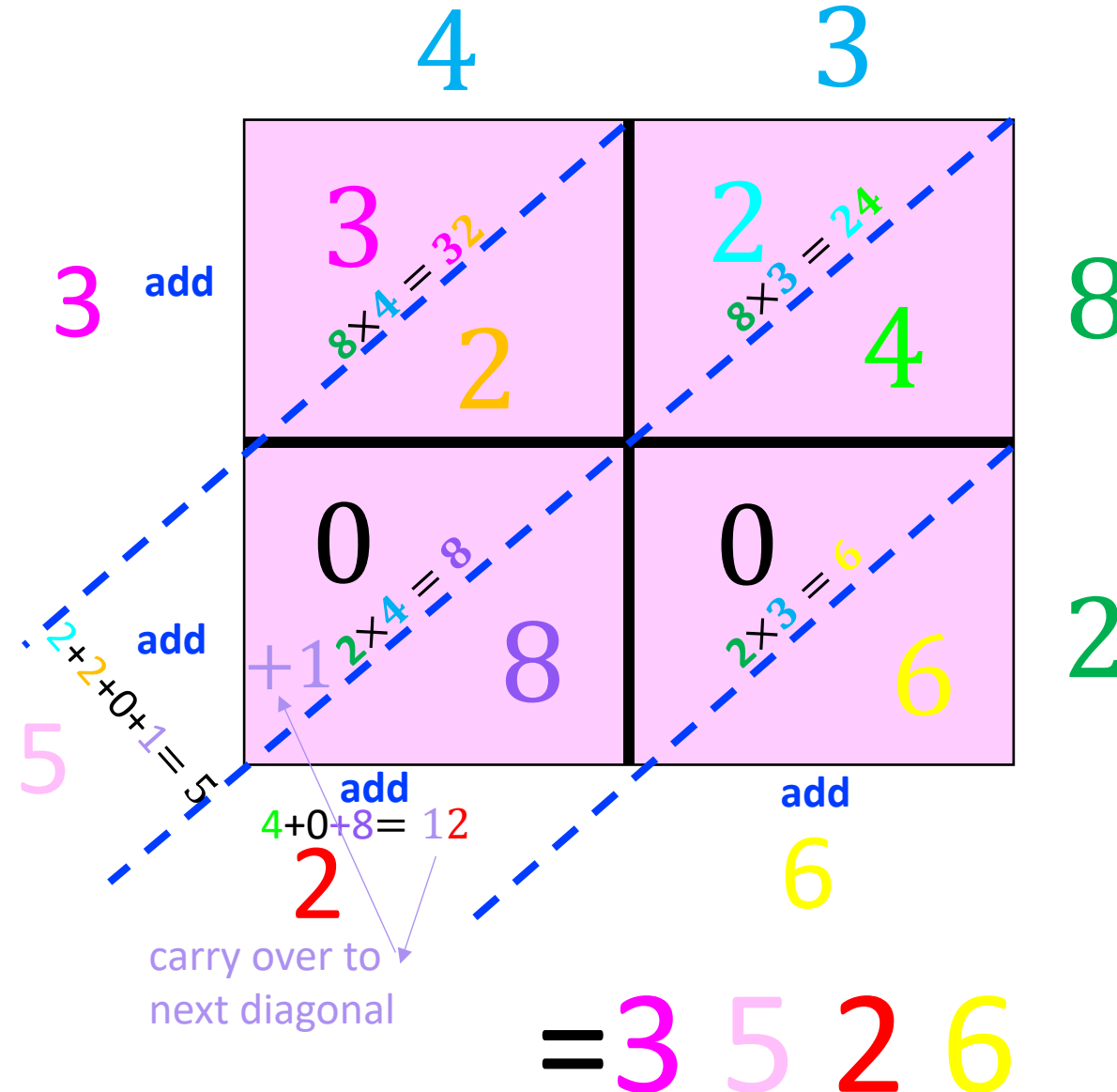
always put a
zero here

Note: This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Way 4

Lattice Method

$$43 \times 82$$



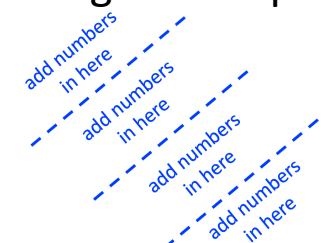
Method:

Step 1:

For each box we FIRST multiply the numbers on the top of the box with the numbers to the far right of the box and THEN split the digits of the number you get from multiplying (this number is shown on top of the diagonal) across the dashed diagonal that divides each box.

Step 2:

Add the numbers in each of the separate diagonal strips



(start on the right). These numbers form our answer (from left to right).

Example 3

$$612 \times 24$$

Way 1

Area Model/Box/Grid Method

$612 = 6$ hundreds (600), 1 tens (10) and 2 ones (2) which means $600 + 10 + 2$

↓ partition and put here

600 10 2

	20×600	20×10	20×2
20	12,000	200	40
	4×600	4×10	4×2
4	2,400	40	8

$24 = 2$ tens (20) and 4 ones (4) which means $20 + 4$

partition and put here

20

4

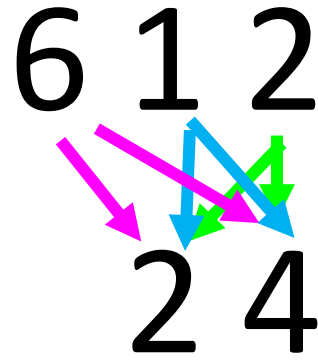
Method

For each box we FIRST multiply the number on the top of the box with the number on the left of the box.

We then add all the numbers in the boxes together.

$$12,000 + 200 + 40 + 2,400 + 40 + 8 = 14,688$$

Way 2



$$2 \times 4 = 8$$

$$2 \times 20 = 40$$

$$10 \times 4 = 40$$

$$10 \times 20 = 200$$

$$600 \times 4 = 2,400$$

$$600 \times 20 = 12,000$$

Method:

Multiply each of the colour pairs and then add the results

$$8 + 40 + 40 + 200 + 2,400 + 12,000 = 14,688$$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1/2)

Step 1

do every multiplication with the pink numbers (carry if we have a two-digit number, just like with addition)

$$\begin{array}{r}
 \text{X} \quad 612 \\
 \hline
 4 \times 6 \quad 4 \times 1 \quad 4 \times 2
 \end{array}$$

we write our answers on the top line

next

Step 2

do every multiplication with the blue numbers (carry if we have a two-digit number, just like with addition)

$$\begin{array}{r}
 \text{X} \quad 612 \\
 \hline
 4 \times 6 \quad 4 \times 1 \quad 4 \times 2 \\
 2 \times 6 \quad 2 \times 1 \quad 2 \times 2 \quad 0
 \end{array}$$

we write our answers on the bottom line

next

Do each calculation and then add

$$\begin{array}{r}
 \text{X} \quad 612 \\
 \hline
 2448 \\
 + 12240 \\
 \hline
 14688
 \end{array}$$

Step 3

always put a zero here

Without all the extra colour coding this looks like:

© mymathscloud

do every multiplication with
the pink numbers
(carry if we have a two-digit
number, just like with addition)

$$\begin{array}{r} 612 \\ \times 24 \\ \hline 2448 \\ \hline \end{array}$$

next \Rightarrow

do every multiplication with
the blue numbers
(carry if we have a two-digit
number, just like with addition)

$$\begin{array}{r} 612 \\ \times 24 \\ \hline 2448 \\ 12240 \\ \hline \end{array}$$

add \Rightarrow

$$\begin{array}{r} 612 \\ \times 24 \\ \hline 2448 \\ + 12240 \\ \hline 14688 \end{array}$$

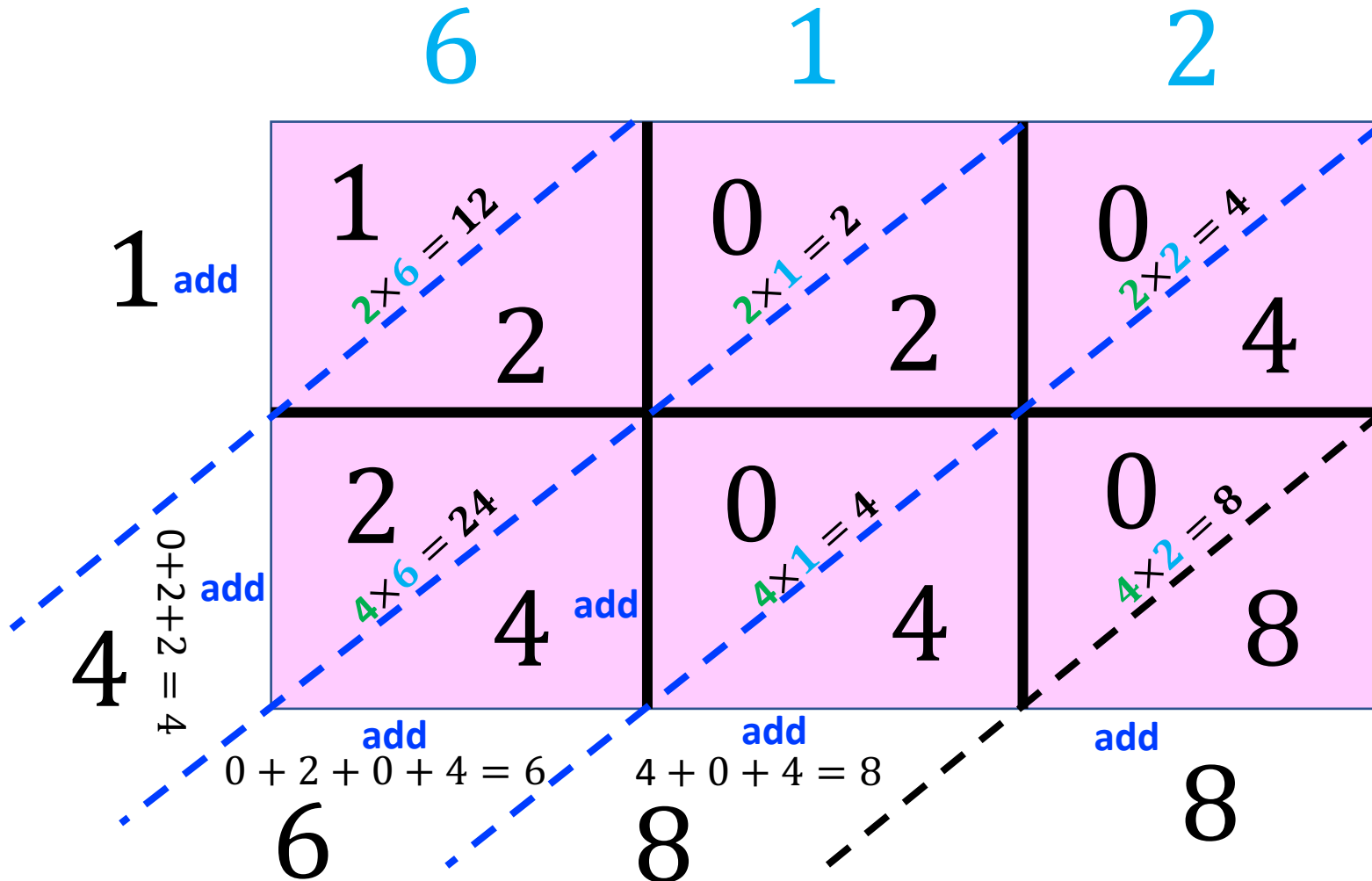
always put a
zero here

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Way 4

Lattice Method

$$612 \times 24$$



© mymathscloud

Method:

Step 1:

For each box we FIRST multiply the numbers on the top of the box with the number on the far right of the box and THEN split the digits of the number you get from multiplying (shown on top of the diagonal across the dashed diagonal that cuts up each box).

Step 2:

Add the numbers in each of the diagonal strips (start on the right). These numbers form our answer (from left to right).

Let's do another example, but this this time only using the most common method which is long multiplication way.
This example is the same as above, except we need to carry more.

Example 4

$$828 \times 35$$

The diagram illustrates the long multiplication process in three stages:

Step 1: Multiplication by the units digit (5)

$$\begin{array}{r} 828 \\ \times 5 \\ \hline \end{array}$$

Calculations and carries: $5 \times 8 = 40$ (write 0, carry 4); $5 \times 2 = 10$ plus carry 4 = 14 (write 4, carry 1); $5 \times 8 = 40$ plus carry 1 = 41 (write 1, carry 4). Result: 4140.

Step 2: Multiplication by the tens digit (3), shifted one place left

$$\begin{array}{r} 828 \\ \times 30 \\ \hline \end{array}$$

Calculations and carries: $3 \times 8 = 24$ (write 4, carry 2); $3 \times 2 = 6$ plus carry 2 = 8 (write 8); $3 \times 8 = 24$ (write 4, carry 2). Result: 24840.

Step 3: Addition of the two partial products

$$\begin{array}{r} 4140 \\ + 24840 \\ \hline 28980 \end{array}$$

The final result is 28980.

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Without all the extra colour coding this looks like:

do every multiplication with
the pink numbers
(carry if we have a two-digit
number, just like with addition)

$$\begin{array}{r}
 \overset{1}{8} \overset{4}{2} 8 \\
 \times \quad 35 \\
 \hline
 4140 \\
 \hline
 \end{array}$$

next



do every multiplication with
the blue numbers
(carry if we have a two-digit
number, just like with addition)

$$\begin{array}{r}
 \overset{2}{8} 2 8 \\
 \times \quad 35 \\
 \hline
 4140 \\
 24840 \\
 \hline
 \end{array}$$

add



$$\begin{array}{r}
 \quad 828 \\
 \times \quad 35 \\
 \hline
 4140 \\
 + 24840 \\
 \hline
 28980
 \end{array}$$

always put a
zero here

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Example 5

$$623 \times 235$$

Way 1

Area Model/Box/Grid Method

600

20

3

200

200×600	200×20	200×3
120,000	4,000	600
30×600	30×20	30×3
18,000	600	90
5×600	5×20	5×3
3,000	100	15

30

5

Method:

For each box we FIRST multiply the number on the top of the box with the number on the left of the box.

We then add all the numbers in the boxes together.

$$120,000 + 4,000 + 600 + 18,000 + 600 + 90 + 3,000 + 100 + 15 = 146,405$$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1)

do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} \overset{1}{6} \overset{1}{2} 3 \\ \times 235 \\ \hline 3115 \end{array}$$

we write our answers on the top line

next

do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} 623 \\ \times 235 \\ \hline 3115 \\ 18690 \end{array}$$

we write our answers on the second line

next

do every multiplication with the purple numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} 623 \\ \times 235 \\ \hline 3115 \\ 18690 \\ + 124600 \\ \hline 146405 \end{array}$$

we write our answers on the third line

always put a zero here

WE always put two zeros here

Way 4

Lattice Method

$$623 \times 235$$

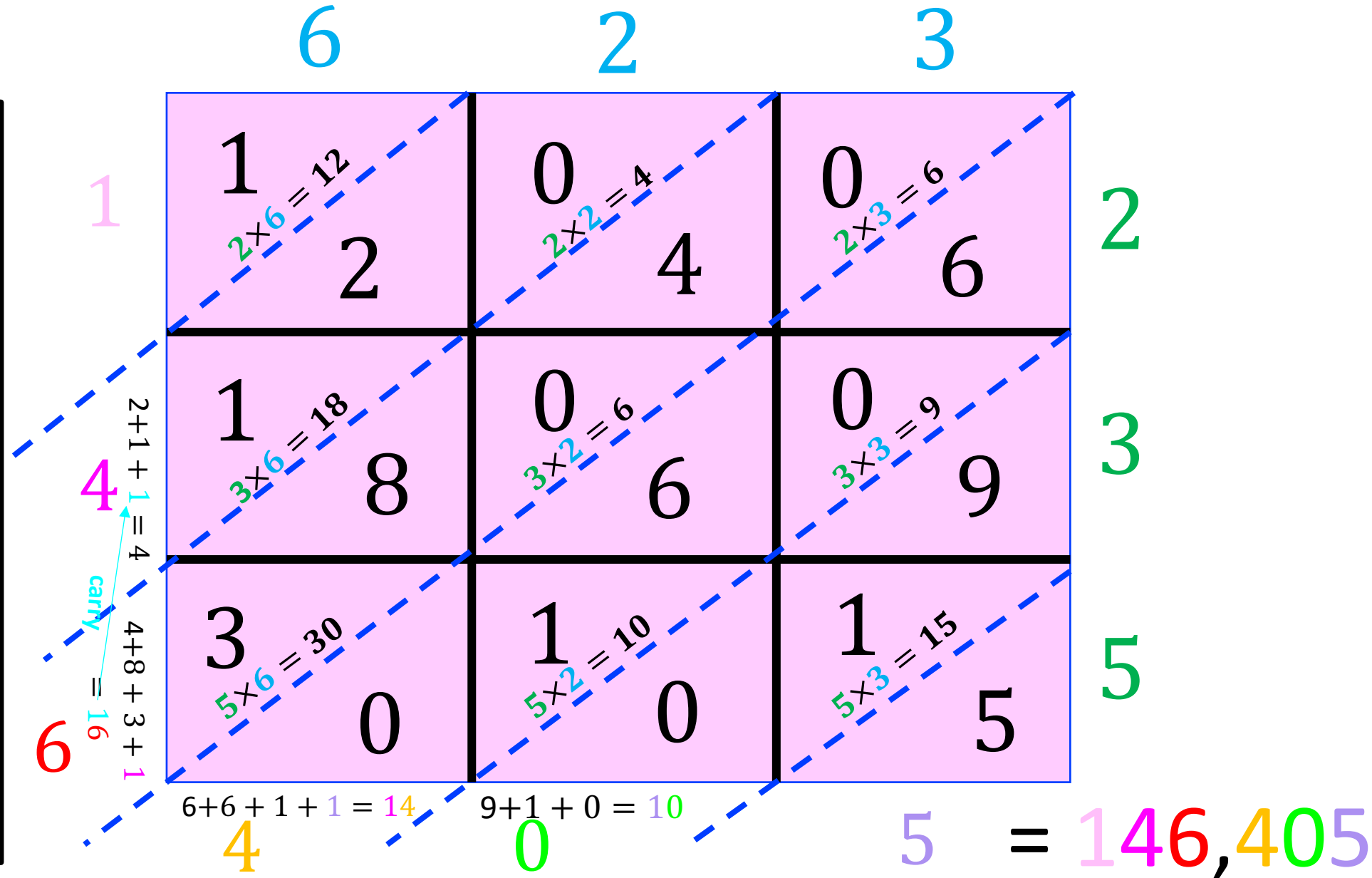
Method:

Step 1:

For each box we FIRST multiply the numbers on the top of the box with the number on the far right of the box and THEN split the digits of the number you get from multiplying (shown on top of the diagonal) across the dashed diagonal that cuts up each box.

Step 2:

Add the numbers in each of the diagonal strips (start on the right). These numbers form our answer (from left to right).



Let's now look at ways 5 and 6

Criss Cross Method

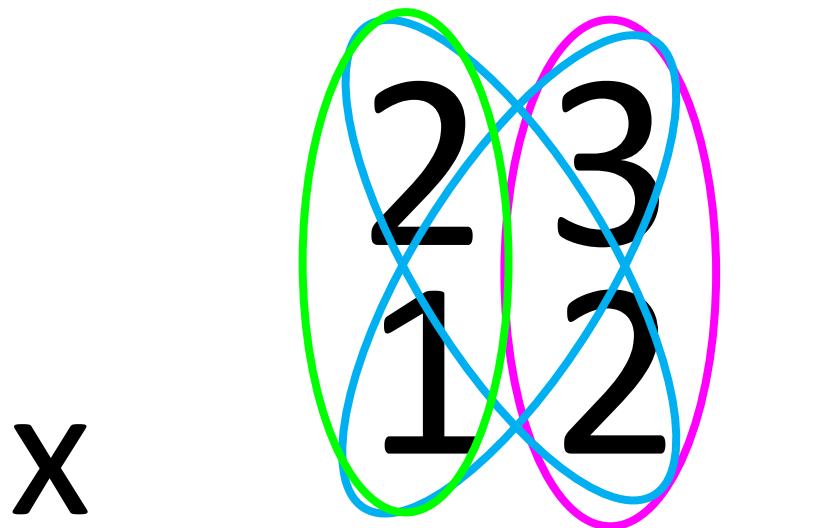
and

Chinese Stick Multiplication

$$23 \times 12$$

Way 5

Criss Cross Method



③ do this last

$$2 \times 1 = 2$$

② do this next

$$2 \times 2 = 4$$

$$1 \times 3 = 3$$

① start here

$$3 \times 2 = 6$$

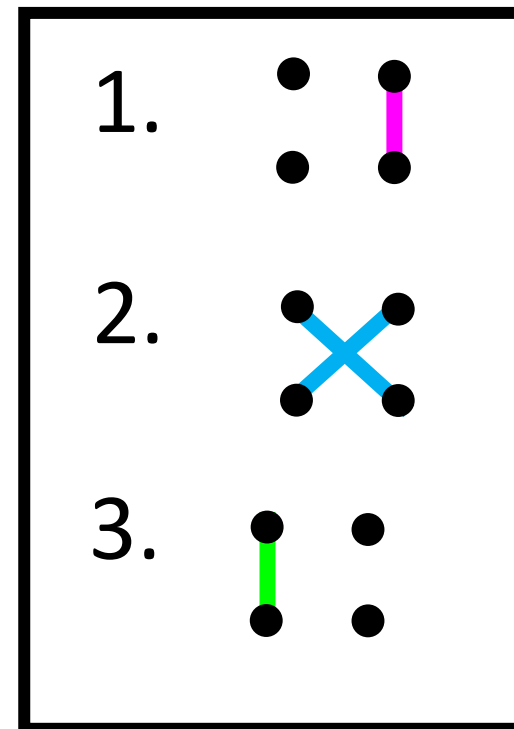
Add these numbers

$$4 + 3 = 7$$

$$= 276$$

Method:

We multiply each of these combinations

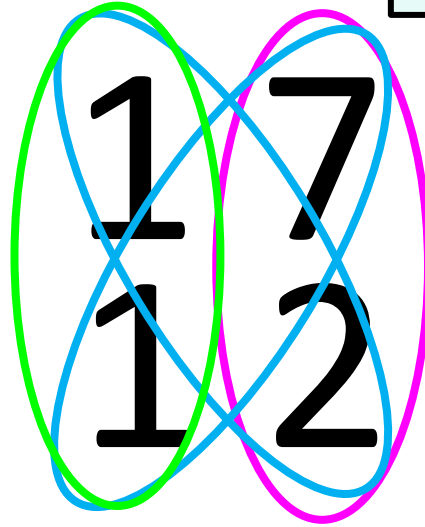


Way 5

Criss Cross Method

$$17 \times 12$$

x



③ do this last

$$1 \times 1 = 1$$

Add

$$1 + 1 = 2$$

② do this next

$$1 \times 2 = 2$$

$$1 \times 7 = 7$$

Add these numbers

$$2 + 7 + 1 = 10$$

we carry the 1 to the next sum

① start here

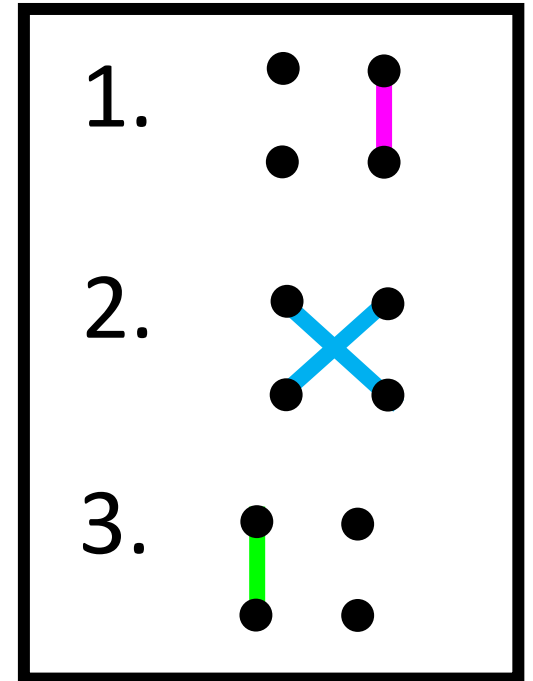
$$7 \times 2 = 14$$

we carry the 1 to the next sum

$$= 204$$

Method:

We multiply each of these combinations



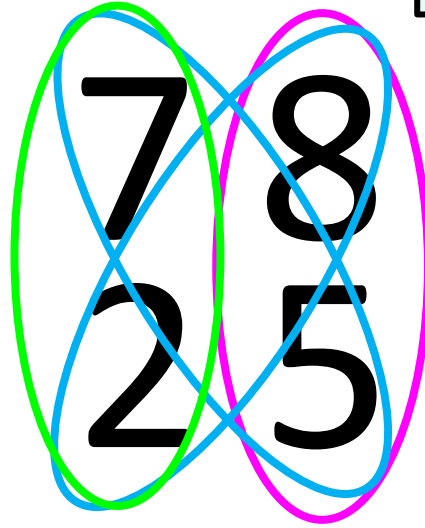
Way 5

Criss Cross Method

$$78 \times 25$$

© mymathscloud

x



③ do this last

$$7 \times 2 = 14$$

Add

$$14 + 5 = 19$$

② do this next

$$7 \times 5 = 35$$

$$2 \times 8 = 16$$

Add these numbers

$$35 + 16 + 4 = 55$$

we carry the 5 to the next sum

① start here

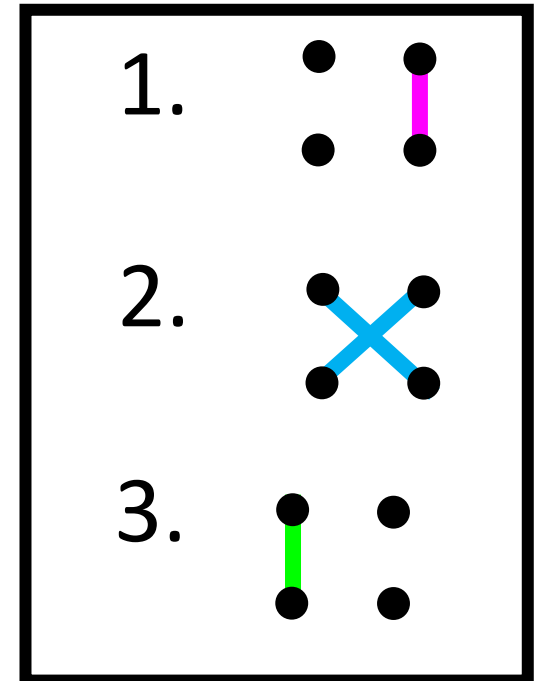
$$8 \times 5 = 40$$

we carry the 4 to the next sum

$$= 1,950$$

Method:

We multiply each of these combinations



Way 5

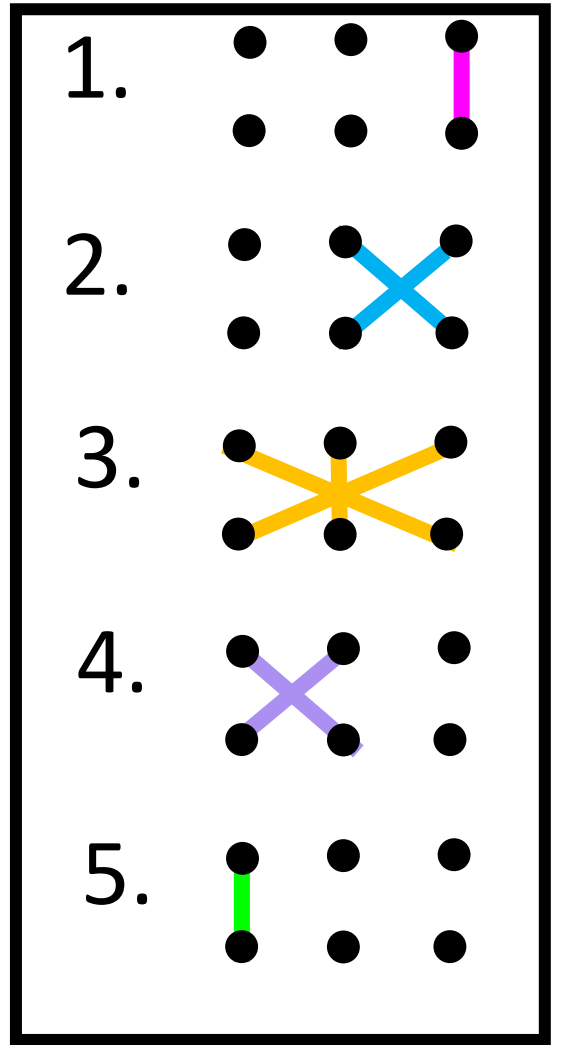
Criss Cross Method

$$123 \times 231$$

$$\begin{array}{r} 123 \\ \times 231 \\ \hline \end{array}$$

Method:

We multiply each of these combinations



⑤ do this last ④ do this fourth ③ do this third ② do this next ① start here

$1 \times 2 = 2$ $1 \times 3 = 3$ $2 \times 3 = 6$ $2 \times 1 = 2$ $3 \times 1 = 3$

$2 \times 2 = 4$ $1 \times 1 = 1$ $3 \times 3 = 9$

Add these numbers $2 \times 3 = 6$ Add these numbers

$3 + 4 + 1 = 8$ Add these numbers $2 + 9 = 11$

$6 + 1 + 6 + 1 = 14$ we carry the 1 to the next sum

we carry the 1 to the next sum

$= 28,413$

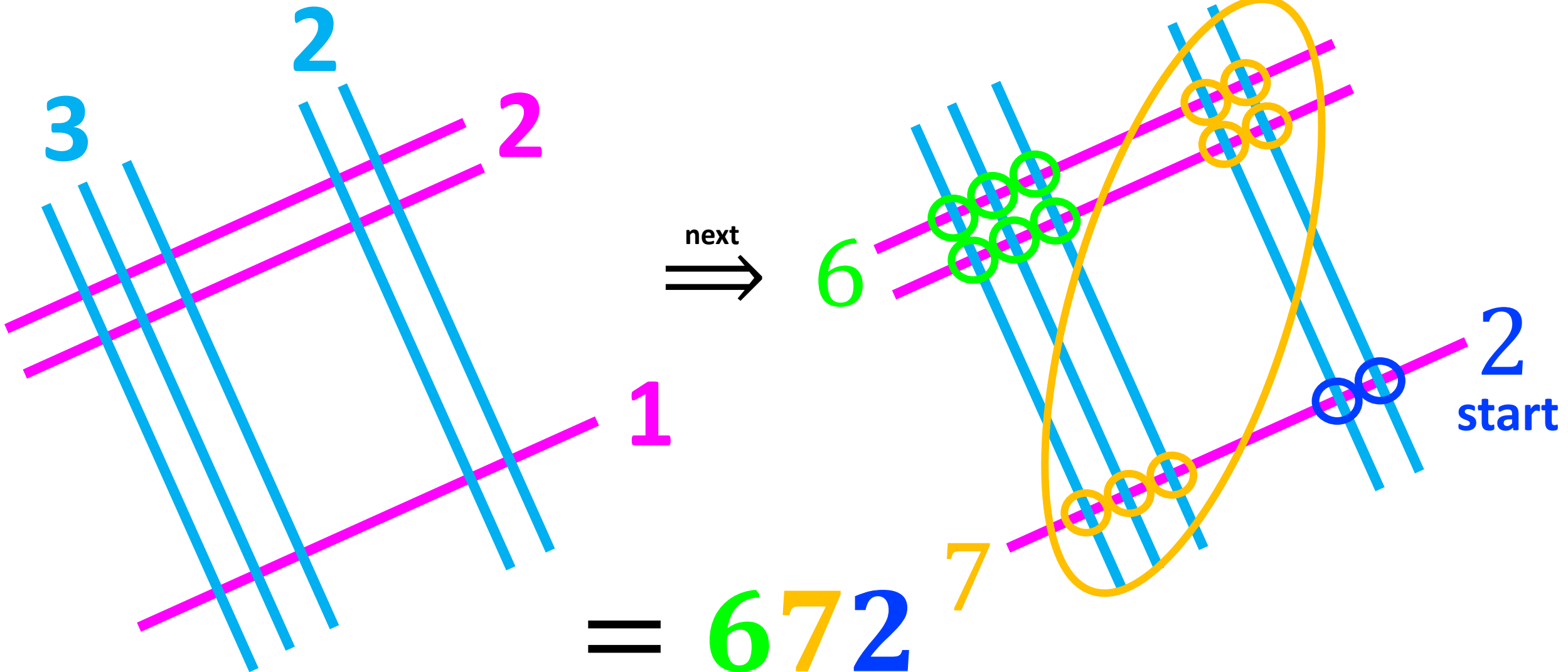
Way 6

Chinese Stick Multiplication

$$21 \times 32$$

$$21 \times 32$$

count the intersections for each colour group



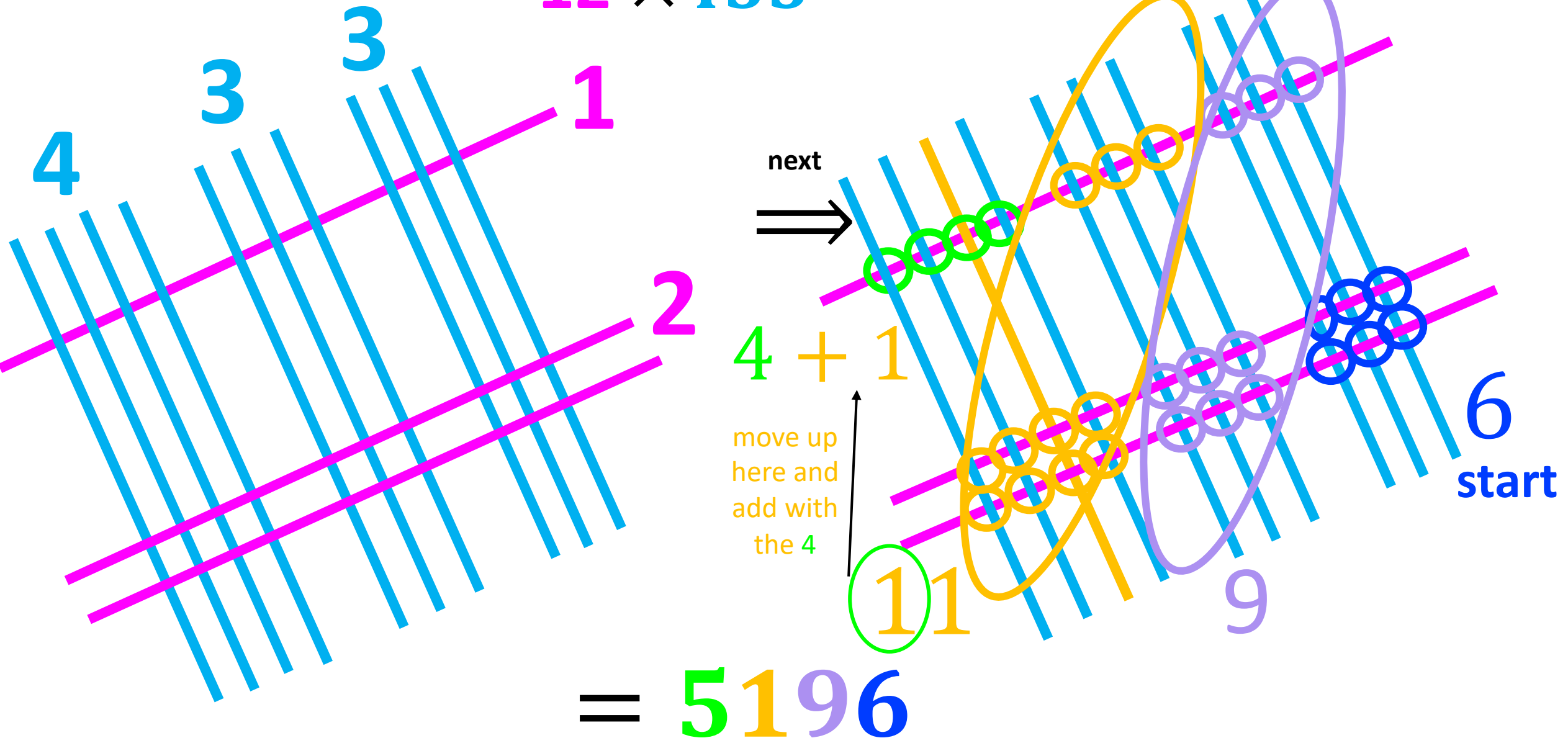
Way 6

Chinese Stick Multiplication

$$433 \times 12$$

$$12 \times 433$$

count the intersections for each colour group

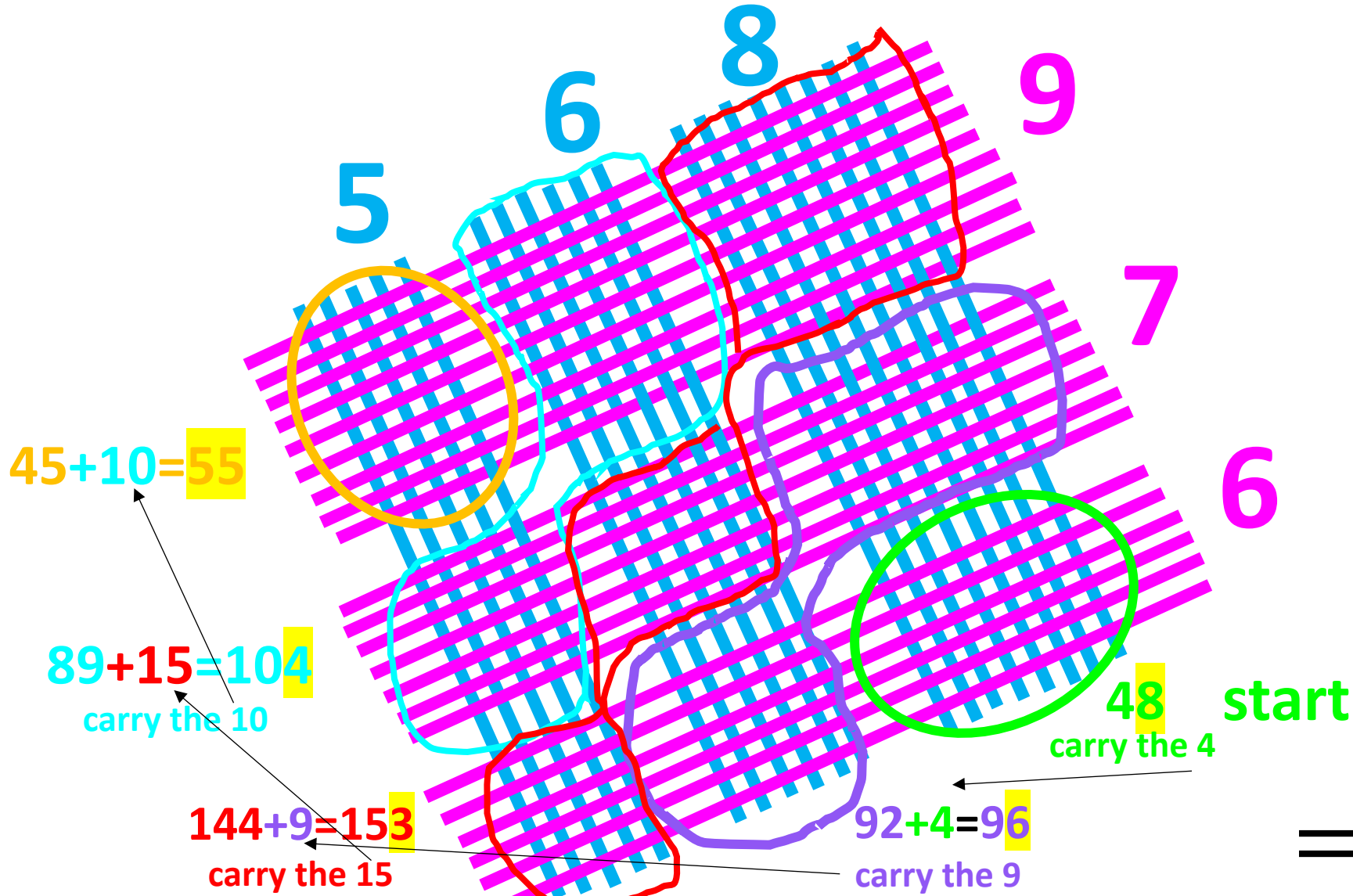


Way 6

Chinese Stick Multiplication

$$568 \times 976$$

$$568 \times 976$$



$$= 554,368$$

Basic Division



The division symbol \div is just a blank fraction, you replace the dots with numbers



$a \div b$ means the same as $\frac{a}{b}$ which is the same as $b \overline{)a}$

Notice that the **numerator** goes underneath the division sign: $\frac{a}{b} = b \overline{)a}$

$$5472 \div 3$$

We now work left to right

Step 1: How many times does the **number** fit into each digit (each colour)

Step 2: Do the calculation to see what the result is

Step 3: Carry the remainder

$$\begin{array}{r}
 1824 \\
 \hline
 3 \overline{)5472}
 \end{array}$$

How many times does 3 fit into 5?

1 time which gives 3 hence has a remainder of 2 (since $5-3=2$)

How many times does 3 fit into 24?

8 times which gives 24 hence no remainder (since $24-24=0$)

How many times does 3 fit into 7?

2 times which gives 6 hence a remainder of 1 (since $7-6=1$)

How many times does 3 fit into 12?

4 times which gives 12 hence no remainder (since $12-12=0$)

$$2274 \div 6$$

We now work left to right

Step 1: How many times does the **number** fit into each digit (each colour)

Step 2: Do the calculation to see what the result is

Step 3: Carry the remainder

$$\begin{array}{r}
 0 \quad 3 \quad 7 \quad 9 \\
 \hline
 6 \overline{) 2 \quad 2 \quad 7 \quad 4}
 \end{array}$$

How many times does 6 fit into 2?

0 times which gives 0 hence has a remainder of 2 (since $2-0=2$)

How many times does 6 fit into 22?

3 times which gives 18 hence a remainder of 4 (since $22-18=4$)

How many times does 6 fit into 47?

7 times which gives 42 hence a remainder of 5 (since $47-42=5$)

How many times does 6 fit into 54?

9 times which gives 54 hence no remainder (since $54-54=0$)

What happens if
the numbers are
bigger?

$$2784 \div 32$$

Option 1

We make the numbers smaller and more manageable (if possible). How we we do this:

$a \div b$ means the same thing as $\frac{a}{b}$ so we are just simplifying a fraction first and then dividing



$$2784 \div 32 = \frac{2784}{32} = \frac{1392}{16} = \frac{696}{8} = \frac{348}{4} = \frac{174}{2}$$

$$\begin{array}{r} 087 \\ 2 \overline{) 174} \\ \underline{14} \\ 87 \end{array}$$

87

Option 2

Divide as normal

$$\begin{array}{r} 0087 \\ 32 \overline{) 2784} \\ \underline{64} \\ 278 \\ \underline{256} \\ 224 \\ \underline{224} \\ 0 \end{array}$$

It is harder to see how many times 32 fits into 278, but it is still doable

What happens if
the number
doesn't fit in
exactly?

$$6281 \div 8$$

Option 1

Divide as usual until you reach the end of the number. We write the remainder at the end

$$\begin{array}{r}
 0785 \text{ remainder } 1 \\
 8 \overline{) 6281} \\
 \hline
 785 \text{ r } 1
 \end{array}$$

Option 2

We put a decimal at the end and carry on by putting zeros for as long as we need (we stop either when the number stops or when we reach our desired accuracy)

$$\begin{array}{r}
 0785.125 \\
 8 \overline{) 6281.100} \\
 \hline
 785.125
 \end{array}$$